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ON DIFFERENTIAL SUBORDINATIONS CONNECTED WITH BAZILOEVIC FUNCTIONS RELATED TO A SECTOR.

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ABSTRACT

In this investigation we generalized the class introduced by J. patel [7] of multivalent analytic functions to the subclass $\omega_p(\lambda,\mu,\sigma,\delta,\mathrm{A},\mathrm{B})$ which has been recently published. Also we derive various properties by using the techniques of Briat-Bouquet differential subordination.

Introduction

Let T(p) dente the class of functions of the form

$$f(z)=z^p+\sum_{n=p+1}^{\infty}a_n z^n \quad (p\in N=\{1,2,\ldots\}) \qquad (1)$$

Which are analytic in the unit disk U={z:|z|<1}.

Let S*p(α) and C*p(α) (0≤ α <p) denote subclass of functions in T(1) which are univalent, star like of order α, convex of order α, respectively. A function f∈ T(p) is said to be multivalent Bazilevic of type μ and order α, if there exists a function $g\in S_p^*$ such that Re

$$\left\{\frac{z\,f'(z)}{f(z)^{1-\mu}g^{\mu}(z)}\right\}\rangle\,\alpha \quad (z\in U) \qquad (2)$$

For some μ (μ ≥ 0) and (0 ≤ α < p). denoted by B(p,μ,α) consider the real numbers A,B such that (-1 ≤ B < A ≤ 1), and

$$S_p^*(A,B)=\left\{f\in T(p):\frac{z\,f'(z)}{f(z)}\langle\; p\,\frac{1+Az}{1+Bz},\ z\in U\right\} \qquad (3)$$

The symbol ($\langle$) stands for subordination.

Let

$$AW_p(\lambda,\mu,\sigma,\delta,\mathrm{A},\mathrm{B})=\left\{\begin{array}{l} f\in T(p):\dfrac{z\,f'(z)}{f(z)^{1-|\sigma|\mu}\,g(z)^{|\sigma|\mu}}+\lambda\left\{\begin{array}{l}1+\dfrac{z\,f''(z)}{f'(z)}-(1-|\sigma|\mu)\dfrac{z\,f'(z)}{f(z)}\\ -|\sigma|\mu)\dfrac{z\,g'(z)}{g(z)}\end{array}\right\}\\ \langle\; p\,\dfrac{1+\{B+(A-B)(1-\delta)\}z}{1+Bz} \end{array}\right\} \qquad (4)$$

for some real μ(μ ≥ 0) , σ ∈ C/ {0} , λ > 0 , 0 < δ ≤ 1 and $g\in S_p^*$.

If σ =1 and δ =0 we have A $W_p(\lambda,\mu,\mathrm{A},\mathrm{B})$ studied by [7].

In order to prove our results, we need the following lemmas:

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Lemma 1: [2] if -1≤ B < A ≤ 1 , β > 0 and $\gamma \in C$ such that R(γ) ≥ $\frac{-B(1-n)}{1-B}$ then differential equation

$$q(z)+\frac{z\,q'(z)}{\beta\,q(z)+\gamma}=\frac{1+Az}{1+Bz} \qquad (z\in U)$$

Has a univalent solution in U given by

$$q(z)=\begin{cases}\dfrac{z^{\beta+\gamma}(1+Bz)^{\frac{\beta(A-B)}{B}}}{\beta\int_d^z t^{\beta+\gamma-1}(1+Bt)^{\frac{\beta(A-B)}{B}}dt}-\dfrac{\gamma}{\beta} & B\neq 0\\[2ex] \dfrac{z^{\beta+\gamma}\exp(BAz)}{\beta\int_0^z t^{\beta+\gamma-1}\exp(BAt)dt}-\dfrac{\gamma}{\beta} & B=0\end{cases} \qquad (5)$$

If Q(z) = 1+c1z + c2z2 + …. is analytic in U and satisfies

$$Q(z)+\frac{z\,Q'(z)}{\beta\,Q(z)+\gamma}\prec\frac{1+Az}{1+Bz} \qquad (z\in U) \qquad (6)$$

$$then \quad Q(z)\prec q(z)\prec\frac{1+Az}{1+Bz} \qquad (z\in U) \qquad (7)$$

And q(z) is the best dominant of (6).

Lemma 2 : Let q be a convex function in U and let $\omega(z)\in U$ be analytic with R{ω(z) ≥ 0 }, if h is analytic in U and h(0) = q(0) then

$$h(z)+\omega(z)zh'(z)\prec q(z) \qquad (z\in U)$$

$$implies \quad h(z)\prec q(z) \qquad (z\in U)$$

Lemma 3: Let v be a positive measure on [0,1], let h be a complex valued function defined on U×[0,1] such that h(.,t) is analytic in U for each t∈[0,1] and h(z,.) is v-integrable on [0,1] for all z∈U in addition, suppose that R{h(z,t)} > 0 , h(-r,t) is real and R{1/h(z,t)}≥ 1/h(-r,t) for all |z|≤ r <1 and t∈[0,1] , if

$$h(z)=\int_0^1 h(z,t)dv(t)$$

then R{1/h(z)}≥ 1/h(-r) (9)

Lemma 4: Let a,b,c∈R and c ≠ 0,-1,-2,… we have

$$\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt=\frac{\Gamma(b)\,\Gamma(-b)}{\Gamma(c)}2F_1(a,b,c,z)(c\rangle b\rangle 0) \qquad (10)$$

2F1(a,b,c,z)=2F1(b,a,c,z) (11)

2 F1(a,b,c,z) = (1-z) 2-a F1(a,c-b,c,z/z-1) (12) (8)

Where r is gamma function and 2F1 is hyper geometric function.

Lemma 5 : Let p(z) = 1+c1+c2z2+… be analytic function is U and p(z)≠0 in U . if there exists apoint z0∈U such that

$$|\arg(p(z))|\prec\frac{\pi}{2}\gamma\,(|z|\langle|z_0|)\ and\ |\arg p(z_0)|=\frac{\pi}{2}\gamma \qquad (0<\gamma\leq 1) \qquad (13)$$

Then

$$\frac{z_0\,p'(z_0)}{p(z_0)}=ik\gamma \quad where \quad \begin{cases}k\geq\frac{1}{2}(x+\frac{1}{x}) \ \ if\ \ \arg p(z_0)=\frac{\pi}{2}\gamma\\ k\leq\frac{1}{2}(x+\frac{1}{x}) \ \ if\ \ \arg p(z_0)=-\frac{\pi}{2}\gamma\end{cases} \qquad (14)$$

And (p(z0)1/γ = ± ix(x > 0) (15) .

Lemma 6 : Let g ∈ S*p(A,B),μ,c are real numbers with μ> o ,

c > max $\left\{ -p\mu, \frac{-p\mu(1-A)}{(1-B)} \right\}$, then $F_{\mu,c} \in S_p^*(A,B)$ (16)

Main Results

Theorem 3.1 : Let f ∈ AWp (λ,μ,σ,δ,A,B) then

$$\frac{z\,p'(z)}{p\,f(z)^{1-|\sigma|\mu}\,g(z)^{|\sigma|\mu}} \langle \frac{\lambda}{pQ(z)} = q(z) \quad (z \in U, \quad -1 \le B \langle A \le 1,\ \lambda \rangle 0, \mu \ge 0, \sigma \in C/\{0\}) \qquad (17)$$

where

$$Q(z)=\begin{cases} \int_0^1 S^{\frac{p}{\lambda}-1}\left(\frac{1+BSz}{1+Bz}\right)^{\frac{p(A-B)(1-\delta)}{\lambda B}} ds & , B \neq 0 \\ \int_0^1 S^{\frac{p}{\lambda}-1} \exp\left(\frac{p}{\lambda}(s-1)(B+(A-B)(1-\delta)z\right)ds & , B = 0 \end{cases} \qquad (18)$$

$$q(z)=\frac{1}{1+Bz} \quad when \quad A=B\left(1-\frac{1}{1-\sigma}-\frac{(1-\delta)\lambda}{p}\right) \qquad , B \neq 0$$

And q is the best dominant of (4). Also , if $A \le B\left(1-\frac{1}{1-\sigma}-\frac{(1-\delta)\lambda}{p}\right)$ with -1≤ B < 0 , then AWp(λ,μ,σ,δ,A,B) C B (p,μ,ξ) , where

$$\xi = p\left\{2F_1\left(1, \frac{p(B-A)(1-\delta)}{\lambda B}; \frac{p}{\lambda}+1; \frac{B}{B-1}\right)\right\}^{-1} \qquad (19).$$

proof :

Assume that $\psi(z)=\frac{z\,f'(z)}{p\,f(z)^{1-|\sigma|\mu}\,g(z)^{|\sigma|\mu}} \quad (z \in U)$ is analytic in U. by logarithmic differentiations the last expression, we get

$$\frac{z\,f'(z)}{f(z)^{1-|\sigma|\mu}\,g(z)^{|\sigma|\mu}} + \lambda\left\{1+\frac{zf''(z)}{f'(z)} - (1-|\sigma|\mu)\frac{zf'(z)}{f(z)} - |\sigma|\mu\frac{zg'(z)}{g(z)}\right\} = p\psi(z) + \lambda\frac{z\psi'(z)}{\psi(z)}$$

$$\langle \frac{p(1+[B+(A-B)(1-\delta)]z)}{1+Bz}$$

There fore ψ(z) satisfies (6) and hence by Lemma (1) ,we have

$$\psi(z) \langle q(z) \langle \frac{1+[B+(A-B)(1-\delta)]z}{1+Bz}$$

where q(z) is given by(5) for A=B+(A-B)(1-δ) , γ = 0 , β = p/λ .

Now if we put $a = \frac{p(B-A)(1-\delta)}{\lambda B}$, c = p/λ +1 , then c > b > 0 for B ≠0 and making use lemma (4) , we have

$$Q(z)=(1+Bz)^{a}\int_{0}^{1}S^{b-1}(1+Bsz)^{-a}\,ds=\frac{\Gamma(b)}{\Gamma(c)}\,2F_{1}(1,a,c,\frac{Bz}{Bz+1}) \quad (20)$$

Our claim R{1/Q(z)} ≥ 1/Q(-1) , z∈U; this implies that inf{Rq(z)}=g(-1) by assumption A < B (1-1/1-δ-(1-δ)λ/p) ,we have c > a > 0 , so by(10) and (20) yields

$$Q(z)=\int_{0}^{1}h(z,s)dv(s)$$

,we have

$$h(z,s)=\frac{1+Bz}{1+(1-s)Bz}$$

(0 ≤ s ≤ 1) and

$$dv=\frac{\Gamma(b)}{\Gamma(a)\Gamma(c-a)}s^{a-1}(1-s)^{c-a-1}ds$$

, the measure of dv(s) > 0 on [0,1]. For -1 ≤ B < 0 , then R{h(z,s)} > 0 h(-r,s) ∈R for 0 ≤ r < 1. and R{1/h(z,s)} = {1+(1-s)Bz/1+Bz} ≥ 1-(1-s)Br/1+1-Br = 1/h(-r,s). Thus by lemma (3), we obtain R{1/Q(z)} ≥ 1/Q(-r) , |z| ≤ r < 1 Let r → 1- we obtain the result. Also by A → (B(1-1/1-δ – (1-δ)λ/p)+ and using (7) , we have AWp(λ,μ,σ,δ,A,B)⊂B(p,μ,ξ) setting μ = 1/|σ| ,

$$A=\frac{p-2\alpha+p\delta}{p(1-\delta)}$$

(p-λ/2≤ α < p) , 0 ≤ δ < 1 and B=-1 in Theorem(3.1), we have.

Corollary (3.1) if f∈T(p) satisfies

$$R\left\{\frac{z f'(z)}{g(z)}+\lambda(1+\frac{2f''(z)}{f'(z)}-\frac{2g'(z)}{g(z)})\right\}\rangle\alpha \qquad (\lambda\rangle 0\ ,\ z\in U)$$

for some $g\in S_{p}^{*}(A,B)$ *,then* $f\in B(\mathrm{T}(p,\lambda,\alpha))$

We have

$$\mathrm{T}(p,\lambda,\alpha,\delta)=p\left\{2F_{1}(1,\frac{p}{\lambda}+\frac{p-2\alpha+p\delta}{(1-\delta)A},\frac{p}{\lambda}+1,\frac{1}{2})\right\}^{-1}$$

setting μ = 0 ,

$$A=\frac{p-2\alpha+p\delta}{p(1-\delta)}.(\frac{p-\lambda}{2}\leq\alpha\langle p)\ \ 0\leq\delta\langle 1\ ,\ B=-1$$

in theorem (3.1) we have Corollary (3.2) if f∈T(p) satisfies

$$R\left\{(1-\lambda)\frac{2f'(z)}{f(z)}+\lambda(1+\frac{2f''(z)}{f(z)})\right\}\rangle\alpha$$

(α > 0 , z∈U)

Then f∈ S*p(T (p,λ,α,δ)) where

$$\tau(p,\lambda,\alpha,\delta)=p\left\{2F_{1}(1,\frac{p}{\lambda}+\frac{p-2\alpha+p\delta}{(1-\delta)\lambda},\frac{p}{\lambda}+1,\frac{1}{2}\right\}^{-1}$$

Theorem (3.2) : Let f∈T(p) such that

$$R\left\{\frac{f(z)}{z^{p}}\right\}\rangle 0\ \ and\ \left|\frac{z f'(z)}{f(z)^{1-|\sigma|\mu}g(z)^{|\sigma|\mu}}-p\right|\langle p$$

(μ ≥ 0 , σ∈C/{0} , z∈U) ; for g∈S then f is multivalent convex in

$$|z|\langle\frac{3+2\mu|\sigma|(p-1)-\sqrt{(3+2\mu|\sigma|(p-1)^{2}-Ap(2\mu|\sigma|p-p-1}}{2(2\mu|\sigma|p-p-1}$$

Proof : Assume that Q˜(z)= $\dfrac{z f'(z)}{p\,f(z)^{1-\mu|\sigma|}\,g(z)^{\mu|\sigma|}}-1$ then q˜(z) is analytic in U, q˜(0) = 0 and |q˜(z)| < 1 for z∈U therefore by schwarz's Lemma we have q˜(z) = z ω(z), we have ω(z) is analytic in U with |ω(z)| ≤ 1 thus

$$z f'(z)=p\ f(z)^{1-\mu|\sigma|}\ g(z)^{|\sigma|\mu}(1+z\omega(z))$$

by logarithmic differentiation in last expression, we have

$$1+\frac{z f''(z)}{f(z)}=(1-\mu|\sigma|)\frac{z f'(z)}{f(z)}+\mu|\sigma|\frac{z g'(z)}{g(z)}+\frac{z(\omega(z)+z\omega'(z))}{1+z\omega(z)}$$

Letting ψ(z) = f(z)/zp = 1+c1z+c2z2+… , R { ψ(z) }> 0 for z∈U then

$$\frac{z f'(z)}{f(z)}=p+\frac{z\psi'(z)}{\psi(z)}$$

$$1+\frac{z f^{//}(z)}{z f^{/}(z)}=(1-\mu|\sigma|)\, p+(1-\mu|\sigma|)\frac{z\psi^{/}(z)}{\psi(z)}+\mu|\sigma|\frac{zg^{/}(z)}{g(z)}+\frac{z(\omega(z)+z\omega^{/}(z))}{1+z\omega(z)}$$

By using the well-known estimate [3]

$$R\left\{\frac{z\psi^{/}(z)}{\psi(z)}\right\}\geq-\frac{2r}{1-r^{2}}\ ,\ R\left\{\frac{zg^{/}(z)}{g(z)}\right\}\geq-\frac{p(1-r)}{1+r}\ \ and\ \ R\left\{\frac{\omega(z)+z\omega^{/}(z)}{1+z\omega(z)}\right\}\geq\frac{-1}{1-r}$$

let $|z| = r < 1$, then

$$R\left\{\frac{z f^{//}(z)}{z f^{/}(z)}\right\}\geq(1-\mu|\sigma|)p+1+\frac{z f^{//}(z)}{z f^{/}(z)}=(1-\mu|\sigma|)\, p+(1-\mu|\sigma|)\frac{-2r}{1-r^{2}}+\mu|\sigma|\frac{-p(1-r)}{1+r}+\frac{-1}{1-r}$$

therefore by assumption we have

$$R\left\{1+\frac{2 f^{//}(z)}{f^{/}(z)}\right\}\geq\frac{(2\mu|\sigma|p-p-1)r^{2}-(3+2\mu|\sigma|(p-1)r+p}{1-r^{2}}$$

>0 then f is multivalent convex.

Setting μ= 0 in above theorem we have f is convex (multivalent) in $|z|\langle\frac{\sqrt{9+4p(p+1)}-3}{2(p+1)}.$ consider the integral transforms of f(z) ∈T(p) by

$$F_{c}(f)(z)=\frac{c+1}{z^{c}}\int_{0}^{z}t^{c-1}f(t)\, dt$$

, Now define Fc,μ,σ as following

$$F_{c,\mu,\sigma}=\left(\frac{c+p|\sigma|\mu}{z^{c}}\int t^{c-1}f(t)^{|\sigma|\mu}dt\right)^{\frac{1}{\mu|\sigma|}}$$

where c,μ are real numbers with μ > 0 , c > -p|σ|μ, σ∈C/{0}.

Theorem 3.3: Let f∈(p). if $\left|\arg\left(\frac{z f^{/}(z)}{f(z)^{1-|\sigma|\mu}g(z)^{|\sigma|\mu}}-\alpha\right)\right|\langle\frac{\pi}{2}\beta\quad(0\leq\alpha\langle p,\ 0\langle\beta\leq1\ )$ for some g∈S*p, then

$$\left|\arg\left(\frac{z(F_{c,\mu,\sigma})^{/}(f)}{F_{c,\mu,\sigma}(f)^{1-|\sigma|\mu}F_{c,\mu,\sigma}(g)^{|\sigma|\mu}}-\alpha\right)\right|\langle\frac{\pi}{2}e$$

where Fc,μσ defined as (1) and (0 < e ≤ 1) is the solution

$$\beta=\begin{cases}e+\frac{2}{\pi}\tan^{-1}\left(\frac{(1+B)e\sin(\pi(1-\mathrm{T}(A,B,c,\mu,\sigma,p))/2)}{(1+B)c+|\sigma|\mu p(1+B+(A-B)(1-\delta))+(1+B)e\ \cos(\pi(1-\mathrm{T}(A,B,c,\mu,\sigma,p))/2}\right) & B\neq-1\\ e & B=-1\end{cases}$$

And

$$\mathrm{T}(A,B,c,\mu,\sigma,p)=\frac{2}{\pi}\sin^{-1}\left(\frac{|\sigma|\mu p(A-B)(1-\delta)}{e(1-B^{2})+|\sigma|\mu p(1-B^{2}-B(A-B)(1-\delta))}\right)$$

$$h(z)=\frac{1}{p-\alpha}\left(\frac{z(F_{c,\mu,\sigma})^{/}(f)}{F_{c,\mu,\sigma}(f)^{1-|\sigma|\mu}F_{c,\mu,\sigma}(g)^{|\sigma|\mu}}-\beta\right)=\frac{H(z)}{E(z)}$$

Proof : Let

Where

$$H(z)=\frac{1}{p-\alpha}\left\{z^{c}f(z)^{|\sigma|\mu}-c\int_{0}^{z}t^{c-1}f(t)^{|\sigma|\mu}dt-|\sigma|\mu\alpha\int_{0}^{z}t^{c-1}g(t)^{|\sigma|\mu}\ dt\right\}$$

and $E(z)=|\sigma|\mu\alpha\int_0^z t^{c-1}g(t)^{|\sigma|\mu}\,dt$, then h(z) is analytic in U and h(0) = H(0) / E(0) = 1. Now

$$H'(z)=\frac{1}{p-\alpha}(cz^{c-1}f(z)^{|\sigma|\mu}+z^{c}|\sigma|\mu f(z)^{|\sigma|\mu-1}f'(z)-cz^{c-1}f(z)^{|\sigma|\mu}-|\sigma|\mu\alpha z^{c-1}g(z)^{|\sigma|\mu})$$

$$=\frac{1}{p-\alpha}(z^{c}|\sigma|\mu f(z)^{|\sigma|\mu-1}f'(z)-|\sigma|\mu\alpha z^{c-1}g(z)^{|\sigma|\mu})$$

$$E'(z)=|\sigma|\mu\alpha z^{c-1}g(z)^{|\sigma|\mu} \quad ,then$$

$$\frac{H'(z)}{E'(z)}=\frac{1}{p-\alpha}\left(\frac{zf'(z)}{f(z)^{1-|\sigma|\mu}g(z)^{|\sigma|\mu}}-\alpha\right), \text{ and by logarithmic differentaitions } h(z) \text{ we have}$$

$$\frac{H'(z)}{E'(z)}=h(z)\left(1+\frac{S(z)}{zS'(z)}\frac{zh'(z)}{h(z)}\right)=\frac{1}{p-\alpha}\left(\frac{zf'(z)}{f(z)^{1-|\sigma|\mu}g(z)^{|\sigma|\mu}}-\alpha\right)$$

By assumption and using lemma (6), we have $F\mu|\sigma|,c(g) \in S^*p$ then

$$|\sigma|\mu\frac{zF'_{c,\mu,\sigma}(g)}{F_{c,\mu,\sigma}(g)}+c=\frac{zS'(z)}{S(z)}=\xi e^{i\pi\varphi/2}$$

$$\text{where } c+\frac{|\sigma|\mu p(1-(B+(A-B)(1-\delta)))}{1-B}\langle\xi\langle c+\frac{|\sigma|\mu p(1+(B(A-B)(1-\delta)))}{1+B}$$

$$-\tau(A,B,c,\mu,\sigma,p)\,\langle\varphi\langle\,\tau(A,B,c,\mu,\sigma,p)$$

$$\text{for } B\neq -1$$

$$\text{and if } B=-1\text{ , we have}$$

$$c+\frac{|\sigma|\mu\, p(2-(A+1)(1-\delta))}{2}\langle\,\xi\,\langle\infty \qquad , \quad -1\langle\,\varphi\,\langle 1.$$

Let ω(z) = S(z)/ z S′(z) in lemma (2), we see that h(z) ≠ 0 in U. suppose that z∘ ∈ U such that |arg h(z)| < π/2 e and |arg h(z∘)| = π/2 e where (∘ < e ≤ 1) and |z| < |z∘| , then by lemma (5) z∘h′(z∘)/h(z∘) = ike.
Now, if h(z∘)1/e = 2x (x > 0), then

$$\arg\left(\frac{z_{\circ}f'(z_{\circ})}{f(z_{\circ})^{1-|\sigma|\mu}g(z_{\circ})^{|\sigma|\mu}}-\alpha\right)=\arg(h(z_{\circ}))+\arg\left(1+\frac{1}{c+\frac{(zF_{c,\mu,\sigma})'(g)(z_{\circ})|\sigma|\mu}{zF_{c,\mu,\sigma}(g)(z_{\circ})}}\cdot\frac{z_{\circ}h'(z_{\circ})}{h(z_{\circ})}\right)$$

$$=\frac{\pi}{2}e+\arg(1+(\xi e^{i\pi/2})^{-1}iek)$$

$$\frac{\pi}{2}e+\tan^{-1}\left(\frac{ek\sin(\pi(1-\varphi)/2}{\xi+\cos(\pi(1-\varphi)/2}\right)$$

$$\geq\frac{\pi}{2}e+\tan^{-1}\left(\frac{e\sin(\pi(1-\mathrm{T}(A,B,c,\mu,\sigma,p)/2)}{c+\frac{|\sigma|\mu p(1+B+(A-B)(1-\delta)}{1+B}+e\cos(\pi(1-\mathrm{T}(A,B,c,\mu,\sigma,p))/2)}\right)$$

$$=\frac{\pi}{2}\beta \qquad \text{where } \quad B\neq -1$$

for case B = -1 , we have $\arg\left(\frac{z f^{/}(z)}{f(z)^{|\sigma|\mu}} - \alpha\right) \geq \frac{\pi}{2} e.$ We conclude that this is a contradiction with our hypothesis the last if h(z∘)1/e = -ix (x > 0) , then

$$\arg\left(\frac{z_{\circ} f^{/}(z_{\circ})}{f(z_{\circ})^{1-|\sigma|\mu} g(z_{\circ})^{|\sigma|\mu}} - \alpha\right) \leq -\frac{\pi}{2} e -$$

$$\tan^{-1}\left(\frac{e \sin(\pi(1-\mathrm{T}(A,B,c,\mu,\sigma,p))/2)}{c + \frac{|\sigma|\mu p(1+B+(A-B)(1-\delta))}{1+B} + e\cos(\pi(1-\tau(A,B,c,\mu,\sigma,p))/2}\right) = -\frac{\pi}{2}\beta$$

where B ≠ -1 , and if B =-1 , we get

$$\arg\left(\frac{z f^{/}(z)}{f(z)^{1-|\sigma|\mu} g(z)^{|\sigma|\mu}} - \alpha\right) \leq -\frac{\pi}{2} e$$

Also contradiction. Setting μ=1/|σ| , B

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دراسة التفاضل الثانوي المرتبط مع دوال باسليوفك المتعلقة بالقطاع الدائري

عبد الرحمن سلمان جمعة علاء عدنان عواد

الخلاصة:

في هذا البحث قمنا بتحقيق المقدمة العامة للدوال التحليلية المتعددة بواسطة [7] Patel J. إلى جزء الصف $\omega_P(\lambda, \mu, \sigma, \delta, A, B)$ حيث كان نشره بشكل جديد وكذلك اشتقاق خواص الدوال المتعددة باستخدام تقنيات بريت للتفاضل الثانوي.