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# High-spin states analysis in 103Mo nucleus with the Interacting Boson- Fermion Model (IBFM)

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**ABSTRACT**

The Interacting Boson-Fermion model (IBFM) has been applied on the neutron-rich 103Mo nucleus for the first time. The IBFM succeeded in describing this nucleus for both energy excitations, electromagnetic and moment properties. The IBFM produces better agreement with experimental results than other theoretical models such as cranked shell model (CSM) and rigid triaxial rotor-plus-particle (RTRP) model. Some new data have been presented for the first time.

## Introduction:

Neutron rich nuclei with A ≥ 100, shows a shape transition from prolate to oblate depending on the filling of the πg9/2 and νh11/2 orbital. The exact nature and location of this transition depends on the interplay between deformation and single particle effects.

Experimentally, initially β-decay studies offered important information on the low-lying excited states of Molybdenum(Mo) isotope, including several lifetime measurements[1-3]. Considerable quadrupole deformation for the unstable neutron-rich Mo isotopes with A> 100 has been deduced experimentally from the measured life times of the first excited states[4]. Latter, γ spectroscopy of fission fragments extended the knowledge on these nuclei[5-8]. The high-spin states in neutron-rich Mo isotopes, which was populated by the 238U(α,f) fusion-fission reaction using the thin- target technique was presented by Hua et al.[7,9]. In these works, they conclude that the h11/2 neutron alignment is responsible for the first band crossing in Mo isotope and the level scheme was extended from spin 31/2+ at 4.215 Mev to spin 39/2+ at 6.309 Mev for 3/2+[411](ground-state) band and the decoupled 5/2-[532] band was extended from spin 35/2- at 4.983 Mev to spin 39/2- at 6.149 Mev, and there is no evidence for blocking in the alignment measured for the νd5/2 band in 103Mo.

Recently, an experiments have been carried out in order to calculate the g factors and the mixing ratios of states excited in secondary fission fragments, following the spontaneous fission of 252Cf [10,11].

The diverse phenomena of nuclear structure in neutron-rich A~ 100 nuclei makes them an ideal testing ground for various theoretical models[12-14]. It has been noticed that the crossing frequency of the aligned band can be reproduced well by calculations using the Cranked shell model[9]. Within the framework of particle-rotor model, the signature splitting observed for the νh11/2 bands is due to the triaxial degree of freedom in the Mo isotopes.

The possible effect of triaxial deformation on the magnetic moments was investigated in the rigid triaxil rotor-plus-particle (RTRP) framework. The calculations suggest that the triaxial deformation plays a strong role in the Mo isotopes. While the low lying energy levels could be reproduced rather well by RTRP model, the same could not be said for the magnetic properties[ 10 ].

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From what has been said in the theoretical overview of the work in this region, it is very clear that this region has given an excellent opportunity for testing the validity of various nuclear models and suitability of two-body interactions.

So, the purpose of the present work is to know whether the Interacting Boson-Fermion Model (IBFM) can produce better results in agreement with the experiments than other theoretical models or not.

**Theory :IBFM**

In the IBFM, odd-A nuclei are described by the coupling of the odd fermionic quasiparticle to a collective boson core[15]. The total Hamiltonian can be written as the sum of three part

$$H = H_B + H_F + V_{BF} \qquad ...(1)$$

where HB is the usual IBM-2 Hamiltonian[16] for the even-even core, HF is the fermion Hamiltonian containing only one-body terms.

$$H_F = \sum_{jm} \varepsilon_j \, \hat{a}_{jm} \, a_{jm} \qquad ............(2)$$

where εj are the quasiparticle energies and a+jm , ajm are the creation (annihilation) operators for the quasiparticle in the eigen state |jm>.

The boson-fermion interaction, VBF that describes the interaction between the odd quasi-nucleon and the even-even core nucleus, has been shown to be dominated by the following three terms[15]:

$$V_{BF} = \sum A_j \, [(\overset{+}{d} \, x \, \tilde{d})^0 \; x \; (\overset{+}{\underset{j}{a}} \, x \, \tilde{a}_j)^0 \;]^0$$
$$+ \sum_{jj'} \Gamma_{jj'} \, [Q^2 \; x((\overset{+}{\underset{j}{a}} \, x \, \tilde{a}_j)^2 \,]^0_0$$
$$+ \sum_{jj'j''} \Lambda^{j''}_{jj'} \, [(\overset{+}{d} \, x \, \tilde{a}_j)^{j''} \; x(\overset{+}{\underset{j}{a}} \, x \, \tilde{d})^{jj'} \,]^0_0 \qquad ........(3)$$

where the core boson quadrupole operator

$$Q^2 = (\overset{+}{s} \, x \, \tilde{d} + \overset{+}{d} \, x \, \tilde{d})^2 + \chi (\overset{+}{d} \, x \, \tilde{d})^2 \qquad ...........(4)$$ and

χ is a parameter shown by microscopic theory to lie through $\pm \frac{\sqrt{7}}{2}$ , s,d, $\overset{+}{s}$ , $\overset{+}{d}$ are

boson operators with $\tilde{a}_{jm}$ = (-1)j-m aj-m and :: denotes normal ordering whereby contributions that arise from commuting the operators are neglected. The first term in VBF is a monopole interaction which plays a minor role in actual calculations. The dominant terms are the second and third, which arise from the quadrupole interaction. The third term represents the exchange of the quasiparticle with one of the two fermion forming a boson and has shown[16] that this exchange force is a consequence of the Pauli principle on the quadrupole interaction between protons and neutrons. The remaining parameters in equation (3) can be related to the Bardeen, Cooper and Schrieffer(BCS)[22] occupation probabilities, uj , vj of the single particle orbits.

The Hamiltonian of equation (1) was diagonalised by means of the standard program ODDA[18] in which the IBFM parameters are identified as: A0 = BFM, Γ0 = BFQ and Λ0 = BFE .

The electromagnetic transition operators can be written as the sum of the two terms, the first of which acts only on the boson part of the wave function, and the second acts only on the fermion part in equation (1) .

In the IBFM the E2 operator is

$$T^{(E2)} = e_B \, Q^{(2)} + e_F \sum Q_{jj'} \, (a_j \, x \, \tilde{a}_{j'})^2 \qquad .............(5)$$

Where eB and eF are the boson and fermion effective charges

The M1 operator is

$$T^{(M1)} = \sqrt{\frac{30}{4\pi}} \, g_B (\overset{+}{d} \, x \, \tilde{d})^{(1)}$$
$$- \sum_{jj'} g_{jj'} \, [\, j(j+1)(2j+1) \, 4\pi \,]^{1/2} \, (a_j \, x \, \tilde{a})^{(1)} \qquad .....(6)$$

Where gB is the boson g-factor determined by the even-even core, and gjj' is the single particle contribution which depends on gland gs (orbital and spin g-factor) of the odd nucleon.

The transition strengths B(E/Mλ) between levels with spin J and J' are obtained from the operators of equation (5) as

$$B(E/M\lambda; J\rightarrow J')=\frac{\left|\langle J'\| T^{(E/M\lambda)}\| J\rangle\right|^2}{(2J+1)} \quad ..........(7)$$

The magnetic dipole moments (μJ) and the electric quadruple moment(QJ) for a state with spin J can be calculated from M1 and E2 operators respectively. From the matrix elements of T(M1) and T(E2) one can

$$\mu_J=\sqrt{\frac{4\pi}{3}}\sqrt{\frac{J}{(2J+1)(J+1)}}\left\langle J\| T^{(M1)}\| J\right\rangle \quad ...........(8)$$

$$Q_J=\sqrt{\frac{16\pi}{5}}\sqrt{\frac{J(2J-1)}{(2J+1)(J+1)(2J+3)}}$$
$$\left\langle J\| T^{(E2)}\| J\right\rangle \quad ...........(9)$$

**Results and Discussion:**

According to the simple shell model the N=50-82 neutron shell contains the 1g7/2, 2d5/2, 3s1/2, 1h11/2 and 2d3/2 orbits which play an active role in excitations of the 103Mo61 nucleus. Since the initial information in most of the experimental works were extracted from the adjacent 104Mo nucleus, so in this work the 103Mo61 is described as a boson 104Mo core losing one neutron from different shell model orbital.

For positive-parity levels calculation, the orbital 2d5/2, 1g7/2 and 2d3/2 were included. For all these orbital with that's used in negative parity states calculation, we performed a BCS(Barden-Cooper-Schrieffer) calculations, which provided the quasiparticle energies(εj) and shell occupancies(υj2) required as input for the IBFM calculations, are listed in table(1).

**Table 1: BCS parameters used for $^{103}$Mo nucleus.**

| | $\varepsilon_j$(Mev) | $\upsilon_j^2$ |
|---|---|---|
| $1g_{7/2}$ | 1.201 | 0.790 |
| $2d_{5/2}$ | 1.311 | 0.413 |
| $2d_{3/2}$ | 1.749 | 0.131 |
| $1h_{11/2}$ | 1.329 | 0.658 |
| $2f_{7/2}$ | 2.648 | 0.223 |
| $1f_{7/2}$ | 3.205 | 0.154 |

The IBFM Hamiltonian was diagonalised by means of the standard program ODDA[18]. The IBFM (boson-fermion interaction strength) parameters, adjusted such as to provide a good description to the experimental excited states, are: BFQ= -0.0028 Mev, BFE= 0.8070 Mev and BFM= -0.1498 Mev. The χ- value is taken to be

-1.323 and OMEGA= 1.747 Mev. The boson core parameters chosen in this work are those reported in ref.[19].

The calculated IBFM positive parity energy spectrum of 103Mo is shown in Fig. 1 in comparison with the experimental data. It was found that the energy levels produced well by the IBFM with 8% only the average percentage deviation from the experimental results.

For the negative- parity states, the only negative parity orbital in this region N= 50-82 is the 1h11/2. From general considerations the high-spin branch ( J≥ 11/2) can be understood as arising from the coupling of the h11/2 orbit to the even-even core states. The experimental negative- parity states extended up to 39/2- at 6.149 Mev. No set of IBFM parameters was found that could reproduce the negative- parity states when using the h11/2 alone. So, the orbital 1f5/2 ( below N= 50) and 2f7/2 ( above N=82) have been added and their BCS parameters are listed in table 1.

The IBFM parameters for negative-parity states are: BFQ= 1.671 Mev, BFE= -2.783 Mev and

BFM= -1.198 Mev. Also, the average percentage deviation between experiment and the IBFM prediction has been found to be 8% only. The calculated IBFM negative parity energy spectrum of 103Mo is shown in Fig. 2 in comparison with the experimental data.

A further step to confirm the IBFM approach could be obtained from a comparison of the electromagnetic properties of the levels and their electromagnetic transition rates.

The effective boson and fermion charges and g-factors used in the calculation of the electromagnetic M1 and E2 transitions were as follows: eB =0.014 eb, eF =0.014 eb, gl = 0, gs = -2.6782 μN and gd = 0.31 μN. These parameters are used for both positive and negative parity states calculation.

Table 2 compares experimental and theoretical prediction branching ratios λ, B(M1)/B(E2) ratios, and mixing ratios δ for all transitions for which this experimental information was available. It has been noticed that the B(M1)/B(E2) ratio increases linearly as the excitation energy increases except for the first transition (see table 2). Calculated and experimental quadrupole and magnetic moments are compared in Table 3. Excellent agreement to the branching, B(M1)/B(E2) , mixing ratios and moments with the available experimental data.

In the RTRP calculations[10] they reduced the effective core 2+ energy, in some cases by as much as 50%, and the coriolis interactions are weakened( which it has effect on the signature splitting), in order to explain both excitation energies and moment properties. No set of parameters was found that could reproduce equally well both the energy spacing and the magnetic moments. They use two different sets of parameters one to reproduce the excitation energies and the other for magnetic moment calculations. While, in the IBFM calculation the same wave function is used for the energy level calculations as well as for electromagnetic properties calculation.

From experimental results[10,11], the band crossing phenomenon in nuclei was studied through the behavior of the moment of inertia according to the rotational frequency.

In this work, our concentration will be on the IBFM prediction whether it is agrees with the experimental results or not. The IBFM calculation shows a rapid increase in the kinematics moment of inertia as the rotational frequency increases for the 3/2+[411] band. For the 5/2-[532] band, it shows different behavior at low rotational frequency ( < 0.35) while they agree at frequencies higher than 0.35 Mev ( Fig. 3). Moreover, unbending behavior in both bands and band crossing has been observed. Almost similar behave has been noticed for dynamic moments of inertia in both experimental and IBFM predication for both bands. It should be mentioned that the moments of inertia were not normalized in this work.

Signature-splitting functions S(I) used is defined as[20]:

$$S(I) = \frac{E(I) - E(I-1)}{E(I) - E(I-2)} \frac{I(I+1) - (I-2))I-1)}{I(I+1) - (I-1)I} - 1$$

Figs.5 and 6 Shows a comparison of the calculated signature splitting with the experiment for 3/2+[411] band and 5/2-[532] band. For 3/2+[411] band the experimental S(I) is significantly smaller than the calculated. Similar deviations of experiment from theory were also noticed in ref.[20]. Since signature splitting can be considered as a consequence of Coriolis coupling, the higher value of the calculated S(I) could be attributed to the IBFM parameters used in the energy levels calculation where the exchange parameter(BFE) and the monopole parameter (BFM) are the dominant parameters than the quadrupole strength parameter. Also, the PRTR calculations[9] indicate that the trend of signature splitting of the νh11/2 orbital in odd-A nuclei is very sensitive to the γ degree of freedom ( S(I) increase with increasing γ value). Moreover, the calculated S(I) is not in the same sign with the experiment S(I) and this can be attributed to the inclusion of the 2d3/2 orbital in the IBFM energy level calculations where the admixture to the j=3/2 will give a S(I) contribution of opposite sign to the other two orbital.

The band Kπ=5/2- based on the 5/2-[532] orbital of the νh11/2 subshell, shows decoupled characteristics and has a large signature splitting ( more larger than 3/2+[411] band ). It has been found that the splitting is larger for 103Mo with smallest neutron number compared to 105,107Mo and even larger than the splitting for 107Ru[21]. The IBFM S(I) result agrees with the experimental result for 103Mo nucleus for this band.

The neutron-rich 103Mo isotope has been studied theoretically. The theoretical model (IBFM) has been tested in analyzing the 103Mo nucleus. The level scheme of the 103Mo was reproduced well by the IBFM for both positive 3/2[411] and negative 5/2[532] bands comparing with experimental and theoretical models.

Electromagnetic properties have been calculated and compared with the available experimental data. The agreement have been found to be better than other theoretical models such as cranked shell model (CSM) and the rigid triaxial rotor-plus-particle (RTRP) model. Some new theoretical data have been reported in the present work for the first time such as λ, B(M1)/B(E2) ratios, δ, QJ and μJ which they were not reported experimentally so far.

Band crossing and unbending moments of inertia have been found from the IBFM calculation which agrees with the experimental conclusion, with little deviation in some cases, although in the present analysis the moments of inertia did not normalized.

The IBFM parameter used, shows a high signature- splitting especially for 3/2+[411] band. Included other orbital from regions N< 50 and N> 82, probably improve the results of the positive parity band.

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Table(2) Comparison between experimental and theoretical values of branching ratios λ, B(M1)/B( E2) ratios, and mixing ratio δ in $^{103}$Mo.

| 2(ji jf) | λ | | | B(M1)/B(E2)($\mu_N^2/e^2b^2$) | | | δ (eb / $\mu_N$) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Exp [10] | RTRP [10] | IBFM | Exp [10] | RTRP [10] | IBFM | Exp [10] | RTRP [10] | IBFM |
| 5+ 3+ | | | 0.24 | | | 3.17 | -0.19(5) | | -0.132 |
| 7+ 5+ | 0.41 (12) | 0.52 | 0.34 | 1.96 (9) | 2.46 | 1.94 | -0.149(29) | -0.207 | -0.133 |
| 9+ 7+ | | | 0.06 | | | 2.41 | | | -0.11 |
| 11+ 9+ | | | 0.39 | | | 4.89 | | | -0.32 |
| 13+ 11+ | | | 0.03 | | | 5.03 | | | -0.094 |

Table(3) Comparison between experimental and theoretical values of Quadrupole and magnetic moments in 103Mo.

| $2J^{\pi}{}_1$ | $Q_J$(eb) | | μ ($\mu_N$) | |
|---|---|---|---|---|
| | Exp | IBFM | Exp [10] | IBFM |
| $3^+$ | | 0.172 | | 0.366 |
| $5^+$ | | -0.100 | 0.143(33) | 0.141 |
| $7^+$ | | -0.158 | -0.11(44) | 0.392 |
| $5^-$ | | -0.279 | | 1.934 |
| $7^-$ | | -0.282 | -0.33(11) | 0.349 |
| $9^-$ | | -0.360 | | 2.620 |
| $11^-$ | | -0.322 | <0 | 0.108 |

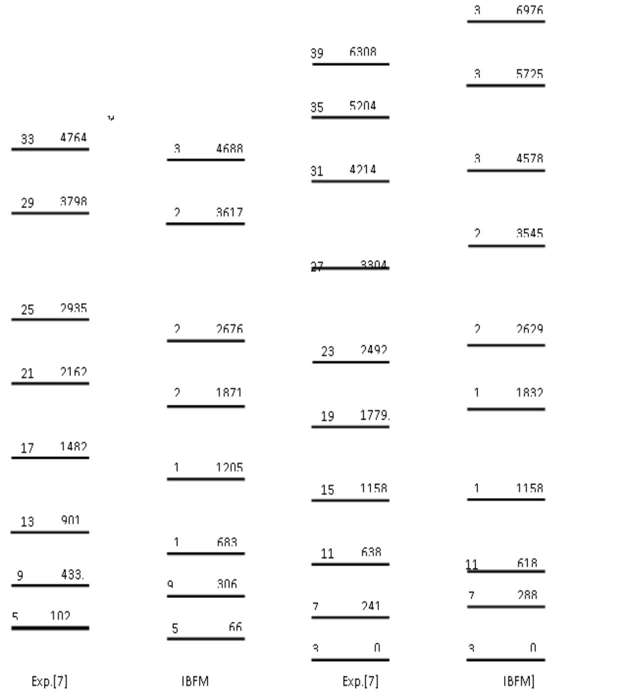


Fig.(1): Comparison of the experimental energy levels for positive parity states in $^{103}$Mo isotope and IBFM prediction. Energies in keV and spins are in 2J.

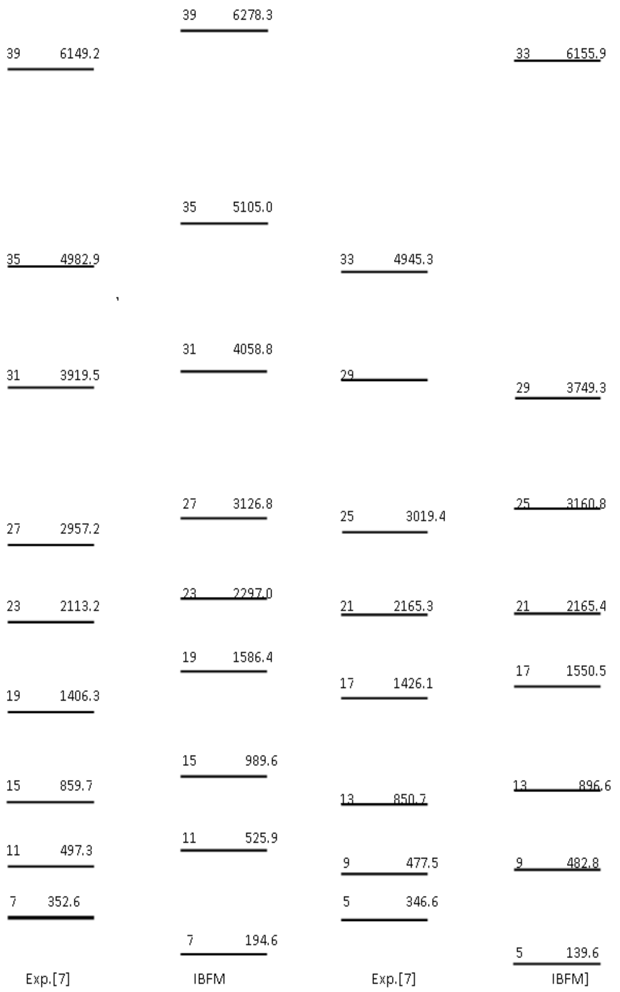


Fig.(2): Comparison of the experimental energy levels for negative parity states in $^{103}$Mo isotope and IBFM prediction. Energies in keV and spins are in 2J.

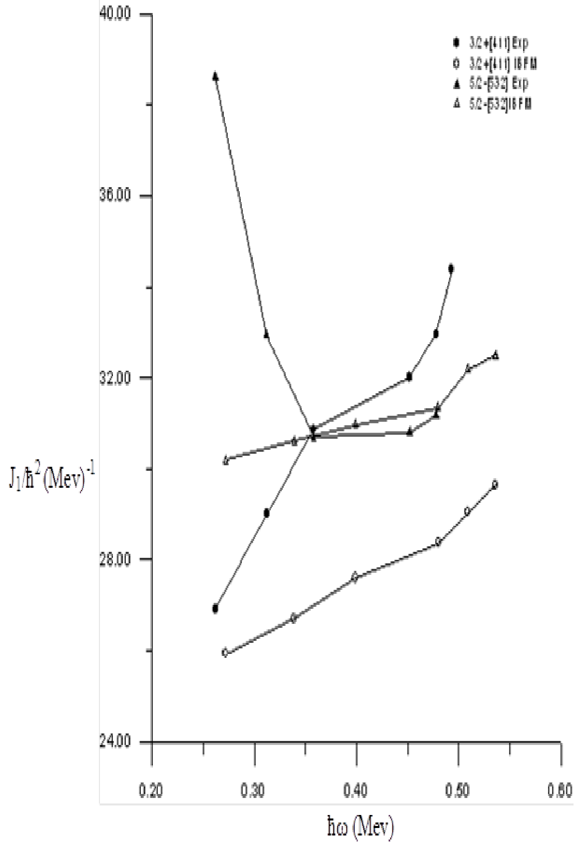


Fig. 3: Comparison between experimental and IBFM prediction of the kinematical moment of inertia as a function of the rotational frequency for $3/2^-$[411] and $5/2^-$[532] bands in $^{103}$Mo.

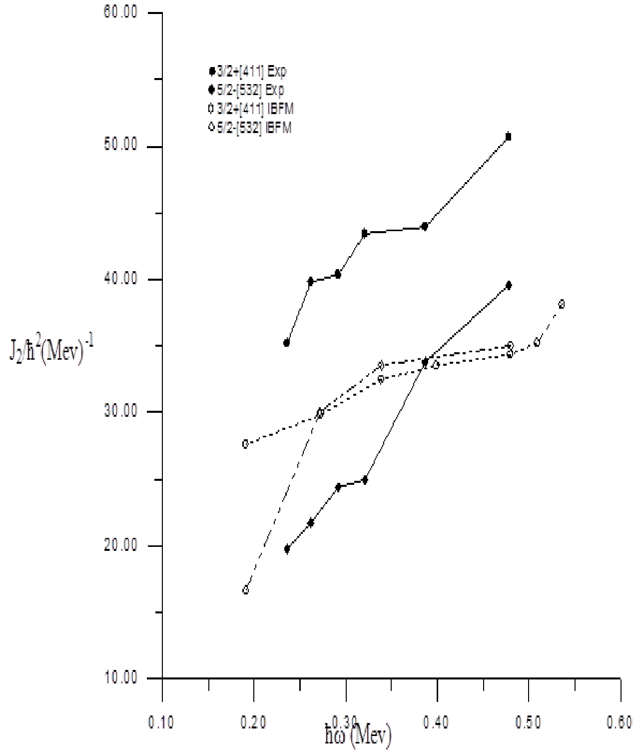


Fig. 4: Comparison between experimental and IBFM prediction of the dynamic moments of inertia as a function of the rotational frequency in $^{103}$Mo nucleus.

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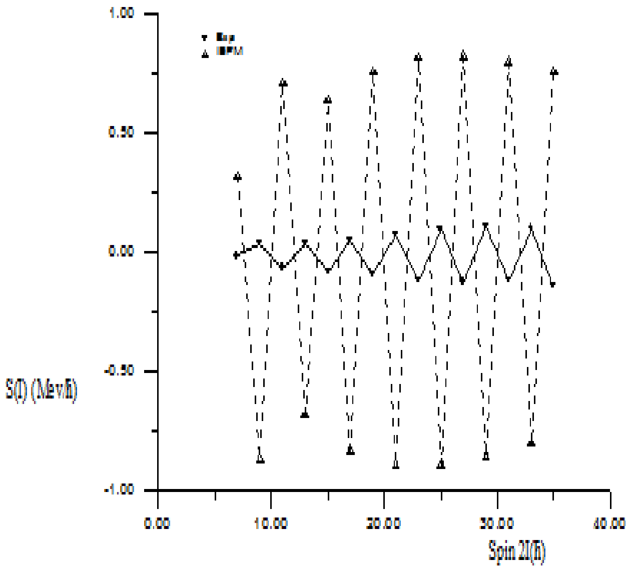


Fig. 5: Experimental and calculated signature splitting for the $3/2^{+}$[411] band as a function of spin in $1^{03}$Mo nucleus.

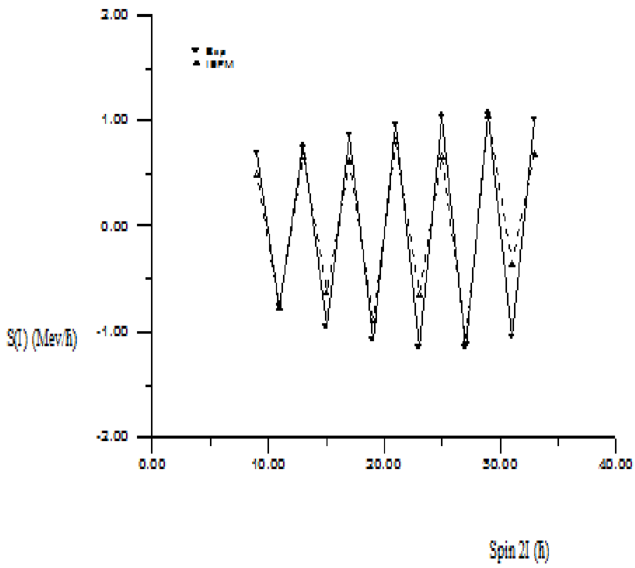


Fig. 6: Experimental and calculated signature splitting for the $5/2^{-}$[532] band as a function of spin in $^{103}$Mo nucleus.

# تحليل مستويات البرم العالي لنواة الـ 103Mo باستخدام نموذج تفاعل البوزون – فرميون


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**الخلاصه:**

تم تطبيق نموذج تفاعل البوزون – فرميون على نواة 103Mo الغنيه بالنيوترونات لاول مره .لقد نجح نموذج تفاعل البوزون – فرميون فى وصف هذه النواة لمستويات الطاقه والخصائص الكهرومغناطيسيه وحساب العزم .أعطى نموذج تفاعل البوزون – فرميون توافق مع النتائج العمليه بشكل أفضل من النماذج النظريه الاخرى مثل نظام القشره التدويرى CSM ونظام الجسم الصلد الثلاثى زائد جسيم RTRP. تم اعطاء بعض المعلومات الجديده لاول مره.