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Mixed Cofibration and Mixed Hurewicz Cofibration

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Abstract: In this papers we study a new concept namely Mixed cofibration (M- cofibration) and Mixed Hurewicz cofibration (M- Hurewicz cofibration).Most of theorem which are valid for cofibrationwill bealso valid for (M- cofibration) the others will be valid if we add extra condition . Among the result we obtain are:
1-A product of two Mixed Cofibration(Mixed Hurewicz cofibration) is also a Mixed Cofibration(Mixed Hurewicz cofibration)2- The M-pullback of Mixed Cofibration(Mixed Hurewicz cofibration)is also Mixed Cofibration(Mixed Hurewicz cofibration)

Keywords: Mixed Cofibration , Mixed Hurewicz Cofibration

Introduction:-

In our papers ,we introduction and study the new concept of M-Cofibration(M-Hurewciz Cofibration) Let Y be any space $f_1:X_1 \to Y$, $f_2:X_2 \to Y$ are two fiber space and $\alpha:X_2 \to X_1$ such that $f_1 o\alpha = f_2$, let $X = \{X_1,X_2\}$, $f = \{f_1,f_2\}$ the $\{X,f,Y,\alpha\}$, has Mixed Lowering homotopy property (M-LHP) w.r.t. a space Z iff given a map $h:Y \to Z$ and a homotopy $g_t:X_1 \to Z$ satisfying $hof_2 = g_0 o\alpha$ then there exist a homotopy $h_t:Y \to Z$ with $h_0 = h$ and $h_t of_1 = g_t$ forall $t \in I$. M-fiber space is called M-cofibration For class $\Re$ if f has (M-LHP) for each $Z \in \Re$.

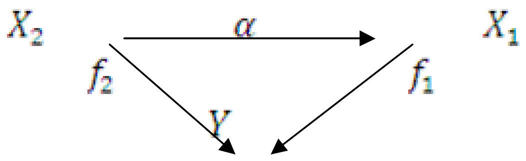
The word map in this work means continuous function, $\Re$ means the classes of topological space and I means [0,1].

Preliminaries:

Record here same basic concepts and clarify notions used in the sequel

Definition(3,2)2-1:- A map $p:E \to B$ is said to have the lowering homotopy property (LHP) w.r.t X iff given a map $h:B \to X$ and a homotopy $f_t:E \to X$ such that $hop = f_0$ then there exist a homotopy $h_t:B \to X$ with $h_0 = h$ and $h_t op = f_t$ for all $t \in I$. Now let $\Re$ be a given class of topological space a map p is a cofibration w.r.t $\Re$ iff $p:E \to B$ has (LHP) w.r.t each $X \in \Re$

Definition(1)2-2:-Let X_1,X_2,Y be three topological space , let $X = \{X_1,X_2\}$, $f = \{f_1,f_2\}$ where $f_1:X_1 \to Y$, $f_2:X_2 \to Y$ are two fiber space and $\alpha:X_2 \to X_1$ such that $f_1 o\alpha = f_2$ then $\{X,f,Y,\alpha\}$ is a M-fiber space (Mixed fiber space)



If $X_1 = X_2 = X$, $\alpha = identity$, $f = f_1 = f_2$ then $\{X, f, Y\}$ is the usual fiber space

2- let $\{X, f, Y, \alpha\}$ be a M-fiber space let $y_0 \in Y$ then $F = \{f(y_0)\}$ is the M-fiber over y_0

Definition(1)2-3:- the $\{X, f, Y, \alpha\}$ be a M-fiber structure , X be any space,and $g: Y' \to Y$ be any continuous map into base Y Let

$X'_1 = \{(x_1, y') \in X_1 \times Y' : f_1(x_1) = g(y')\}$ and

$X'_2 = \{(x_2, y') \in X_1 \times Y' : f_2(x_2) = g(y')\}$ then

$\underline{X'} = \{X_1', X_2'\}$ is called a M-pullback of $\underline{f}$ by g and

$\underline{f'} = \{f_1', f_2'\} : \underline{X'} \to Y'$ is called induced M-function of $\underline{f}$ by g

Define $\alpha' : X_2' \to X_1$ by

$\alpha'(x_2, y') = (\alpha(x_2), y')$

To show that α' is continuous

Since $\alpha' = \alpha \times I_{y'}$, α is continuous and $I_{y'}$ is continuous then α' is continuous

To show α' is commutative

$f'_1 o\alpha'(x_2, y') = f'_1(\alpha(x_2), y') = y'$

$f'_2(x_2, y') = y'$ .therefore $f'_1 o\alpha' = f'_2$

M-Cofibration

Definition 3-1:-Let Y be any space $f_1 : X_1 \to Y$, $f_2 : X_2 \to Y$ are two fiber space and $\alpha : X_2 \to X_1$ such that $f_1 o\alpha = f_2$, let $X = \{X_1, X_2\}$, $f = \{f_1, f_2\}$ the $\{X, f, Y, \alpha\}$, has Mixed Lowering homotopy property (M-LHP) w.r.t. a space Z iff given a map $h: Y \to Z$ and a homotopy $g_t : X_1 \to Z$ satisfying $hof_2 = g_0 o\alpha$ then there exist a homotopy $h_t : Y \to Z$ with $h_0 = h$ and $h_t of_1 = g_t$ forall $t \in I$. M-fiber space is called M-cofibration For class $\Re$ if f has (M-LHP) for each $Z \in \Re$

Proposition 3-2:-Every Cofibration is Mixed Cofibration

Proof:- let $\{X, f, Y, \alpha\}$ be a M-fiber space such that $X_1 = X_2 = X$, $\alpha = identity$, $f = f_1 = f_2$. let $h: Y \to Z$ and a homotopy $g_t : X_1 \to Z$ such that $hof_2 = g_0 o\alpha$ then there exist a homotopy $h_t : Y \to Z$ with $h_0 = h$ and $h_t of_1 = g_t$ forall $t \in I$

Then f has (M-LHP) w.r.t Z

Therefore f has M-cofibration

Proposition 3-3:- let $\underline{f} : \underline{X} \to Y$ and $\underline{f}' : \underline{X}' \to Y'$ be two M-Cofibration then $\underline{f} \times \underline{f}' : \underline{X} \times \underline{X}' \to Y \times Y'$ is also M-Cofibration

Proof:- let Z be any arbitrary space

Let $h^* : Y \times Y' \to Z$ be map where $h: Y \to Z$ and $h' : Y' \to Z$ and

Define $g_t^* : X_1 \times X'_1 \to Z$ as $h^* o(f_2 \times f_2') = g_0^* o(\alpha \times \alpha')$ such that

$g_t' : X'_1 \to Z$ and $g_t : X_1 \to Z$ .since $\underline{f}, \underline{f}'$ are M- Cofibration

Then there exist a homotopy $h_t : Y \to Z$ with $h_0 = h$ and $h_t of_1 = g_t$

and a homotopy $h_t':Y'\to Z$ with $h_0'=h'$ and $h_t'of_1'=g_t$

Now for $h_t^*:Y\times Y'\to Z$ define as $h_t^*o(f_1\times f_1')=g_t^*$ and $h_0^*=h^*$

Therefore $\underline{f}\times\underline{f}':\underline{X}\times\underline{X}'\to Y\times Y'$ is M-Cofibration

Proposition 3-4:-The M-pullback of M- C0fibration is also M-C0fibration

Proof:- let $h':Y'\to Z$ and $h:Y\to Z$ .Defin a homotopy $g_t:X_1\to Z$ such that $hof_2=g_0o\alpha$. since f has M-cofibration then there exist a homotopy $h_t:Y\to Z$ with $h_0=h$ and $h_tof_1=g_t$

Defin $g_t':X'_1\to Z$ such that $h'of_2'=g_0'o\alpha'$ and $g_t'=g_toL$

then there exist a homotopy $h_t':Y'\to Z$ with $h_0'=h'$ and $h_t'of_1'=g_t$

Therefore $\underline{f}':\underline{X}'\to Y'$ has M-cofibration

M- Hurewicz Cofibration

Definition 4-1:- the $\{X,f,Y,\alpha\}$ be a M-fiber structure over Y ,we say that $\underline{f}$ is M-Hurewicz Cofibration iff f has (M-LHP) w.r.t all spaces

Proposition4-2:- let $\underline{f}:\underline{X}\to Y$ and $\underline{f}':\underline{X}'\to Y'$ be two M –Hurewicz Cofibration then $\underline{f}\times\underline{f}':\underline{X}\times\underline{X}'\to Y\times Y'$ is also M-Hurewicz Cofibration.

Proof:- let Z be any arbitrary space

Let $h^*:Y\times Y'\to Z$ be map where $h:Y\to Z$ and $h':Y'\to Z$ and

Define $g_t^*:X_1\times X'_1\to Z$ as $h^*o(f_2\times f_2')=g_0^*o(\alpha\times\alpha')$ such that

$g_t':X'_1\to Z$ and $g_t:X_1\to Z$ .since $\underline{f},\underline{f}'$ are M- Hurewicz Cofibration

Then there exist a homotopy $h_t:Y\to Z$ with $h_0=h$ and $h_tof_1=g_t$

and a homotopy $h_t':Y'\to Z$ with $h_0'=h'$ and $h_t'of_1'=g_t$

Now for $h_t^*:Y\times Y'\to Z$ define as $h_t^*o(f_1\times f_1')=g_t^*$ and $h_0^*=h^*$

Since Z be any arbitrary

Therefore $\underline{f}\times\underline{f}':\underline{X}\times\underline{X}'\to Y\times Y'$ is M-Hurewicz Cofibration

Proposition 4-3: The M-pullback of M-Hurewicz Cofibration is also M-Hurewicz Cofibration

Proof:- let Z be any arbitrary space, let $h':Y'\to Z$ and $h:Y\to Z$ .Defin a homotopy $g_t:X_1\to Z$ such that $hof_2=g_0o\alpha$. since f has M- Hurewicz cofibration then there exist a homotopy $h_t:Y\to Z$ with $h_0=h$ and $h_tof_1=g_t$

Defin $g_t':X'_1\to Z$ such that $h'of_2'=g_0'o\alpha'$ and $g_t'=g_toL$

then there exist a homotopy $h_t':Y'\to Z$ with $h_0'=h'$ and $h_t'of_1'=g_t$

since Z be any arbitrary space

Therefore $\underline{f}':\underline{X}'\to Y'$ has M- Hurewicz cofibration

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اللاتليفات –M(المختلطة) واللاتليفات هريوز - M (المختلطة)

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الخلاصة:

في هذا البحث درسنا مفهوم جديد اسمه اللاتليفات- M(المختلطة) واللاتليفات هريوز -M (المختلطة) التي يرمز لهـــا (M- cofibration) و (M- Hurewicz cofibration).

معظم النظريات الصادقة في اللاتليفات تكون صادقة في اللاتليفات المختلطة واللاتليفات هريوز المختلطـــة اذا أضـــفنا عليها بعض الشروط . وعلية حصلنا على النتائج التالية:-

1. جداء اللاتليفين– M(اللا تليفين هريوز–M) يكون اللا تليف– M (اللا تليف هريوز – M)
2. السحب الخلفي- M الى اللا تليف – M(اللا تليف هريوز – M) يكون اللا تليف – M (اللا تليف هريوز – M)