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FP-Modules and Related Concepts

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ABSTRACT

In this work, we introduce the concept of FP-Module as a generalization of the concept Q-Module. Many characterizations and properties of FP-Modules are obtained. We investigate conditions for FP-Modules to be Q-Modules. Modules which are related to FP-Modules are studied. Some classes of modules which are FP-Modules are given. Furthermore, characterizations of FP-Modules in some classes of modules are obtained.

Introduction

Throughout this paper, R will denote an associative ring with identity, and all R-modules are unitary (left) R-modules. An R-module M is called a Q-Module, if every submodule of M is a quasi-injective [12]. An R-module M is called a quasi-injective, if for each submodule N of M and each R-homomorphism from N into M can be extended to an R-homomorphism from M into M [9]. An R-module M is called a pseudo-injective, if for each submodule N of M and each R-monomorphism from N- into M can be extended to an R-homomorphism from M into M. For an R-module M, E(M) stand for the injective envelope of M. A submodule of an R-module M is called a fully invariant if $f(N) \subseteq N, for\ each\ f \in End(M)$ [18]. An R-module M is called uniform, if every submodule of M is essential in M, where we said that a submodule N of M is essential in M if $N \cap K \neq (0)$ for each submodule K of M. which is equivalent to say that $0 \neq m \in M$, there exists $0 \neq r \in R$ such that $0 \neq mr \in N$. [6].

§1 Basic properties of FP-Modules

In this section, we introduce the definition of FP-Module and give examples characterizations and some basic properties of this concept.

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Definition 1.1

An R-module M is called a FP-Module, if every submodule of M is a finitely pseudo-injective.

Examples and Remarks 1.2

1. Every submodule of FP-Module is a FP-Module.
2. A direct summand of FP-Module is FP-Module.
3. Z_n as a Z-module is FP-Module for every n
4. Every simple R-module is FP-Module.
5. Z_{p^∞} as a Z-module is FP-Module.
6. Z as a Z-module is not a FP-Module, and Q as a Z-module is a quasi-injective, but not a FP-module

1. The inverse image of FP-Module is not necessary FP-Module. For example the Z-module Z_2 is a FP-Module and if we let $f: Z \to Z_2$ defined by $f(x) = \begin{cases} o, & if\ x\ is\ even \\ 1, & if\ x\ is\ odd \end{cases}$

It is clear that f is Z-homomorphism and $f^{-1}(Z_2) = Z$ is not a FP-Module.

1. The direct sum of two FP-Modules is not necessary FP-Module. For example the Z-modules Z_2 and Z_4 are FP-Modules, but $Z_2 \oplus Z_4$ is not FP-Module, (since $Z_2 \oplus Z_4$ itself is not finitely pseudo-injective Z-module.)
2. If M is FP-Module, then $M \oplus M$ is not necessary FP-Module. For example, since Z_4 as a Z-module is FP-

Module, but $Z_4 \oplus Z_4$ is not FP-Module.

Before we give the main result of this section we introduce the following lemma.

Lemma 1.3

Any fully invariant submodule of a finitely pseudo-injective module is a finitelypseudo-injective.

Proof

Let K be a fully invariant submodule of finitely pseudo- injective module M, let L be any submodule of K, and $f: L \to K$ be any R-monomorphism. Since M is a finitely pseudo injective, then there exists an R-homomorphism $g: M \to M$ such that g extend f But K is a fully invariant submodule of M, then $g(K) \subseteq K$. Let $g|K = h: K \to K$. Then for all $x \in K, h(x) = g(x) = f(x)$. That is h is extends f.

Hence K is a finitely pseudo injective. ■

Theorem 1.4

Let M be an R-module. Then the following statements are equivalent.

1. M is a FP-Module.
2. M is afinitely pseudo-injective and every essential submodule of M is a fully invariant under monomorphisms of $End_R(M)$.
3. Every essential submodule of M is afinitely pseudo-injective.

Proof

$(1) \Rightarrow (2)$ Let N be an essential submodule of M , then N is afinitely pseudo-injective . Let $f: M \to M$ be an R- monomorphism and $K = \{x \in N: f(x) \in N\}$, that is $K = f^{-1}(N)$. Since N is a finitely pseudo-injective, then there exists $g: N \to N$ which extends g. We claim that $(h - f)(N) = (0)$. Suppose that $(h - f)(N) \neq (0)$, then $(h - f)(N) \cap N \neq (0)$, for N is an essential submodule of M, which implies that $(h - f)(n) = l$ for some n , l in N. Thus $(h - f)(n) = l$ implies that $(g - f)(n) = l$, then $f(n) = g(n) - l \in N$. This shows that $n \in K$. so $(h - f)(n) = (0)$ which is contradicts the assumption, hence $(h - f)(N) = (0)$ implies that h(N)=f(N). But f(N)=h(N)=g(N)$\subseteq$N. then f(N)$\subseteq$N.And since M is FP-Module then M is finitely pseudo – injective .

$(2) \Rightarrow (3)$ Let N be an essential submodule of M. Then by hypothesis N is is a fully invariant under monomorphism of $End_R(M)$. Hence by Lemma 1.3 N is a finitely pseudo-injective.

$(3) \Rightarrow (1)$ Let N be a submodule of M, then $N \oplus C$ is an essential submodule of M, which implies that N is an essential submodule of M [8]. Hence M is a FP-Module. ■

Now, we look at the injective hull of FP-Module. It turns out that under certain condition it's also FP-Module.

Proposition 1.5

Let M be a FP-Module such that every submodule of E(M) is isomorphic to subquotient of M. Then M is FP-Module if and only if E(M) is a FP-Module.

Proof $\Rightarrow$ Let N be a submodule of E(M). Then N is isomorphic to a subquotient of M. Hence by [10] N is a submodule of M. therefore N is a finitely pseudo-injective.

$\Leftarrow$ trivial. ■

§2 Relationships between FP-Modules and finitely pseudo-injective modules

It's clear that every FP-Module is a finitely pseudo-injective, but the converse is not true (see Example and Remarks 1.2 (6).). In the following propositions, we give conditions under which finitely pseudo-injective modules become FP-Modules.

Recall that an R-module M is duo module if every submodule of M is a fully invariant [18].

Proposition 2.1

Let M be duo module. Then M is a FP-Module if and only if M is a finitely pseudo-injective.

Proof:

Let N be a submodule of M, then N is a fully invariant submodule of M. Hence by lemma 1.3 N is afinitely pseudo-injective. Therefore M is a FP-

Module. ■

Recall that an R-module M satisfies Bears Criterion, if every submodule of M satisfies Baer criterion, where we say that a submodule N of M satisfies Bears Criterion, if for each R-homomorphism $f: N \to M$ there exists r in R such that $f(n) = rn, \forall n \in N$ [1].. ■

Proposition 2.2

Let M be an R-module which satisfies Bears criterion. Then M is FP-Module if and only if M is a finitely pseudo-injective.

Proof

Let N be a submodule of M, then N satisfies Baer's criterion. Hence N is a fully invariant submodule of M (since for each $f \in End(M)$, and fore each $n \in N, f(n) = rn \in N \text{ for some } r \in R$). Hence by lemma 1.3 N is a finitely pseudo-injective. Therefore M is a FP-Module.

Recall that a submodule N of an R-module M is annihilator, if $N = ann_M(I)$ for some ideal I of R [14].. ■

Proposition2.3

Let M be an R-module in which all its submodules are annihilator. Then M is FP-Module if and only if M is a finitely pseudo-injective.

Proof

Let N be a submodule of M, then N is an annihilator submodule. That is $N = ann_M(I)$ for some ideal I of R. We claim that N is a fully invariant submodule of M. Let $f \in End(M)$, then $0 = f(IN) = If(N). Hence\ f(N) \subseteq ann_M(I) = N$. Thus N is a fully invariant submodule of M. Therefore by Lemma 1.3 N is afinitely pseudo-injective. Hence M is FP-Module. . ■

Proposition2.4

Let M be an R-module such that every cyclic submodule of M is fully invariant. Then M is FP-Module if and only M is a finitely pseudo-injective.

Proof

Let N be a submodule of M. Since every cyclic submodule of M is a fully invariant in M, then for each $f \in End(M)$ and for each x in N, $f((x)) \subseteq (x) \subseteq N. Thus\ f(x) \in N$. Hence N is a fully invariant submodule of M. Thus by Lemma 1.3 N is a finitely pseudo- injective. Hence M is FP-Module. ■

Recall that a submodule N of an R-module M is closed, if N has no proper essential extension. [6]

Proposition2.5

Let M be an R-module, such that every submodule of M is closed. Then M is a FP-Module if and only if M is a finitely pseudo- injective.

Proof

Let N be submodule of M, then N is a closed submodule of M. Since M is a finitely pseudo- injective, then by [4, Cor.1.3] N is a direct summand of M, and by [8, Lemma 1] N is afinitely pseudo- injective. Hence M is FP-Module. ■

Since a direct summand of any module is closed [6] we get the following.

Corollary 2.6

Let M be an R-module, such that every submodule of M is a direct summand. Then M is a FP-Module if and only if M is afinitely pseudo- injective.

Recall that a submodule N of an R-module is quasi-stable if for every submodule K of M with K⊆N and every R-homomorphism g: $K \to M$ such that $Img \subseteq N$, then $h(N) \subseteq N$ for each R-homomorphism h: $N \to M$ such that $g = h \circ i_K$.[1].

§3: Relationships between FP-Modules and Q-Modules

In this section we study the relation between FP-Modules and Q-Modules.

Since every quasi-injective module is a pseudo injective hence finitely pseudo-injective, but the converse is not true [9], then every Q-Module is FP-Module but the converse is not true. Thus under certain conditions FP-Module become Q-Modules.

Proposition3.1

Let M be an R-module over a principle ideal domain. Then M is a Q-Module if and only if M is FP-Module.

Proof

Let N be a submodule of M. Since M is an R-module over a principle ideal domain, then N is a submodule over a principle ideal domain. But M is a FP-Module, and then N is a finitely pseudo injective. Thus by [15, Th.3.3] N is a quasi-injective. Hence M is a Q-Module. ■

It is given in [15, Cor. 3.9] that any torsion free module which is finitely pseudo- injective is a quasi-injective, we get the following proposition.

Proposition 3.2

Let M be torsion free R-module. Then M is a Q-Module if and only if M is FP-Module.

Proposition 3.3

Let M be a torsion module over quasi-Dedekind ring. Then M is a Q-Module if and only if M is FP-Module.

Proof

Let N be a submodule of M, then N is afinitely pseudo injective module over quasi-Dedekind ring. Since M is torsion module, then N is a torsion submodule. Thus by [16, Th. 2] N is a quasi-injective. Hence M is a Q-Module.■

The following proposition shows that over a generalized uniserial ring, FP-Modules and Q-Modules are equivalent.

Proposition 3.4

Let M be an R-module over a generalized uniserial ring R. Then M is a Q-Module if and only if FP-Module.

Proof

Let N be submodule of M, then N is afinitely pseudo injective submodule over a generalized uniserial ring R. Hence by [8, Th.4] N is a quasi-injective. Therefore M is a Q-Module.■

Proposition3.5

Let M be a uniform non-singular module. Then M is a Q-Module if and only if M is FP-Module.

Proof

Let N be a submodule of M. Since M is a uniform, then N is a uniform, also , since M is a non-singular, then by [6] N is a non-singular. Let L be a submodule of N and $f: L \to N$ be an R-homomorphism, then since N is non-singular, uniform, so $Ker f = (0) or\ Ker f = L$. If $Ker f = L$, then f can be trivially extended to a homomorphism from N into N. If $Ker f = (0)$, then f is monomorphism and form finitely pseudo-injectivity of N, f can be extended to an R-homomorphism from N into N. hence N is a quasi-injective and then M is a Q-Module.■

It is well-known a finitely pseudo- injective torsion module over a multiplication ring or hereditary ring is a quasi-injective [16, Cor.1].

We end this section by the following result.

Proposition 3.6

Let M be a torsion module over a multiplication ring or hereditary ring R. Then M is a Q-Module if and only if M is FP-Module.

§4 Modules imply FP-Modules

In this section we establish modules which imply Fp-Modules. Recall that an R-module M is a semi-simple, if every submodule of M is a direct summand [6].

The following proposition shows that semi-simple modules imply FP-Modules

Proposition 4.1

If M is a semi-simple R-module, then M is a FP-Module.

Proof

Since M is semi simple R-module ,then M isQ-Module by [12].Hence M is FP-Module

.

The converse of prop. 4.1 is not true in general .In fact the Z-moduleZ_8 is FP-Module, but not semi-simple.

The following proposition gives a condition under which FP-Modules semi simple Modules.

Proposition 4.2

If M is FP-Module such that every submodule of M is a closed, then M is a semi-simple.

Proof

Let N be a submodule of M. Then by hypothesis N is closed. Since M is FP-Module, then M is a finitely pseudo-injective. Therefore by [4, Cor. 13] N is a direct summand of M. Hence M is a semi-simple.■

From proposition 2.5, proposition 4.1 and proposition 4.2, we get the following result.

Proposition 4.3

Let M be an R-module such that every submodule of M is a closed. Then the following statements are equivalent.

1. M is a semi-simple module.
2. M is FP-Module.
3. M is a finitely pseudo-injective module.

Recall that an R-module M is anti-hopfain if every proper submodule of M is a non-hopf kernel. Where, a submodule N of M is called a non-hopf kernel if there exists an isomorphism between M/N and M [7].

It is well-known that anti-hopfain module, is a quasi-injective (pseudo-injective hence finitely pseudo-injective) [2]. Also every submodule of anti-hopfain module is anti-hopfain [2] we get the following results.

Proposition 4.4

If M is an anti-hopfain R-module, then M is FP-Module.

Corollary 4.5

If M is an anti-hopfain R-module, then M/N is FP-Module for any submodule N of M.

The following proposition shows that the homomorphic image of anti-hopfain module is Fp-Module.

Proposition 4.6

If M is an anti-hopfain R-module, then f(M) is FP-Module for each R-homomorphism $f: M \to M'$ Where M' is any R-module.

Proof

Suppose that M is an anti-hopfain module and $f: M \to M'$ be an R-homomorphism. Thus $M/kerf \cong f(M)$. Since M is an anti-hopfain, then by Corollary 4.5 $M/kerf$ is P-Module. Hence f(M) is FP-Module.■

§5 FP-Modules and Multiplication modules

An R-module M is called multiplication module, if every submodule of M is of the form IM for some ideal I of R [3].

In this section we study the relation of multiplication modules with FP-Modules.

We preface our section by the following theorem which gives the relationship between FP-Modules over R and FP-Modules over $End_R(M)$.

Theorem 5.1

If M is a multiplication module, then M is FP-Module over R if and only if M is FP-Module over S where$S = End_R(M)$.

Proof

$(\Rightarrow)$ Let N be S-submodule of M. Since M is a multiplication, then N is an R-submodule of M, then N is afinitely pseudo-injective submodule of M. Hence M is FP-Module over S.

$(\Leftarrow)$ Let N be R-submodule of M . Since M is a multiplication, then by [13, Prop. 1.1] N is an S-submodule of M. Then N is afinitely pseudo-injective submodule of M. Hence M is FP-Module over R. ■

In the following theorem we give a characterization of FP-Module in class of multiplication modules.

A submodule N of an R-module M is called a quasi-invertible if $Hom\left(\frac{M}{N}, M\right) = (0)$ [11].

Theorem 5.2

Let M be a multiplication module with $ann_R(M)$ is a prime ideal of R. Then M is FP-Module if and only if every quasi-invertible submodule of M is a finitely pseudo-injective.

Proof

$(\Rightarrow)$ Trivil..

$(\Leftarrow)$ Let N be a submodule of M. Then $N \oplus K$ is an essential submodule of M, where K is an intersection relative complement of N in M. We claim that $N \oplus K$ is a quasi-invertible submodule of M. Let $f \in Hom(M/N \oplus K, M), f \neq 0$. Thus, there exists an element $m + (N \oplus K) \in M/N \oplus K$ such that $f\big(m + (N \oplus K)\big) = y \neq 0, y \in M$. Since $N \oplus K$ is an essential submodule of M, then there exists a non zero element r in R such thatrm $\neq (0) \in N \oplus K$. Hence $0 = rf(m + N \oplus K) = ry$ and hence $r \in ann_R(y)$ .Since M is multiplication module then by[5, Prop.1] $Ry = IM$ for some ideal I of R. Thus $0 = rIM$ and hence $rI \subseteq ann_R(M)$. Since $ann_R(M)$ is a prime ideal of R, then either $I \subseteq ann_R(M)$ or $r \in ann_R(M)$. If $I \subseteq ann_R(M), then\ IM = Ry = (0)$ and hence $y = 0$, and this is a contradiction. If $r \in ann_R(M)$, then $rm = 0$, for all m in M, this is a contradiction a gain. Thus $f \in Hom(M/N \oplus K, M)$ must be zero. Hence $Hom(M/N \oplus K, M) = (0)$, which implies that $N \oplus K$ is a quasi-invertible submodule of M. Then by hypothesis $N \oplus K$ is a finitely pseudo-injective submodule of M. Hence by [8, lemma1] N is a finitely pseudo-injective submodule of M. Therefore M is FP-Module.■

As an immediate consequence of Th.5.2 we have the following result.

Corollary 5.3

Let M be a prime multiplication module. Then M is FP-Module if and only if every a quasi-invertible submodule of M is a finitely pseudo-injective.

Proposition 5.4

If M is a finitely pseudo-injective multiplication module, then M is a FP-Module.

Proof

Let N be a submodule of M. Since M is a multiplication module then $N = IM$ for some ideal I of R. Let $f \in End_R(M)$, then $f(N) = f(IM) = If(M) \subseteq IM = N$. Hence N is a fully invariant submodule of M. Since M is a finitely pseudo-injective, therefore by lemma 1.3 N is a finitely pseudo-injective. Thus M is FP-Module. ■

The following corollary is an immediate consequence of Prop. 5.4.

Corollary 5.5

If M is a cyclic multiplication module, then M is FP-module.

§6 Characterizations of FP-Modules in some types of modules.

Definition 6.1

An R-module M is called a pseudo-duo module, if every submodule of M is a fully invariant under monomorphisms of $End_R(M)$.

Proposition 6.2

Let M be a uniform module, then M is FP-Module if and only if M is a finitely pseudo-injective and pseudo due module.

Proof

$(\Rightarrow)$ Since M is FP-Module, then M is a finitely pseudo-injective. Let N be a submodule of M. Since M is a uniform module, then N is essential submodule of M. Hence by Theorem 1.4 N is a fully invariant under monomorphisms of $End_R(M)$. Therefore, M is a finitely pseudo-duo module.

$(\Leftarrow)$ Let N be a submodule of M. Since M is a uniform module, then N is an essential submodule of M. And since M is pseudo-duo module, then N is fully invariant under a monomorphism $End_R(M)$. Now, every essential submodule is fully invariant under monomorphism of $End_R(M)$. Hence by Theorem 1.4 M is FP-Module.■

Recall that an R-module M is a monoform , if every non-zero homomorphism $f \in Hom(N, M)$(where N is any submodule is a monomorphism [17].

It is well-known that a uniform module is a uniform we get the following immediate consequence of prop. 6.2.

Corollary 6.3

Let M be a monoform module. Then M is FP-Module if and only if M is a finitely pseudo-injective and pseudo-duo.

Recall that an R-module M is a rational extension of an R-submodule N of M, provided that $Hom_R\left(\frac{K}{N}, M\right) = (0), whenever\ N \subseteq K \subseteq M$. [6]

Proposition 6.4

Let M be a rational extension of every submodule of M. Then M is FP-Module if and only if M is a finitely pseudo-injective and pseudo-duo module.

Proof

$(\Rightarrow)$ Since M is a FP-Module, then M is a finitely pseudo-injective module. Let N be a submodule of M.

Since M is a rational extension of N, then clearly is an essential submodule of M, then by Theorem 1.4 N is a fully invariant under monomorphisms of $End_R(M)$. Hence M is a finitely pseudo-duo module.

$(\Leftarrow)$ Let N be a submodule. Since M is a rational extension of N, then N is an essential submodule of M. And since M is a finitely pseudo-duo module, then N is a fully invariant under a monomorphisms of $End_R(M)$. Hence by Theorem 1.4 M is FP-Module.■

The following theorem gives many characterization of FP-Module in class of a non-singular modules.

Theorem 6.5

Let M be a non-singular R-module. Then the following statements are equivalent.

1. M is a P-Module.
2. Every a quasi-invertible submodule of M is a pseudo-injective.
3. Every dense submodule of M is a pseudo-injective.

Proof

$(1) \Rightarrow (2)$ Trival.

$(2) \Rightarrow (3)$ Let N be a dense submodule of M. Since M is a non-singular, then by [10] N is an essential submodule of M. We claim that N a quasi-invertible submodule of M. Let $g \in Hom_R\left(\frac{M}{N}, M\right), g \neq 0$, thus there exists $x \in M$ such that $g(x+N) = m \neq 0, where\ m \in M.$ Let $r \in R\ and\ r \notin ann(m)$.Hence $rm \neq 0\ and\ rx \notin N.$ Since N is an essential submodule of M, then there exists a non-zero element $s \in R\ such\ that\ srx$ is a non-zero element of N. thus $0 = g(srx+N) = srg(x+N) = srm$, this implies that $sr \in ann(m)$. Therefore $ann(m)$ is an essential ideal of R. Since M is non-singular, then $m = 0$ and hence $g = 0$. Therefore $Hom_R(M/N, M) = (0)$ which implies that N is a quasi-invertible submodule of M. Hence by hypothesis N is a finitely pseudo-injective.

$(3) \Rightarrow (1)$ Let N be a submodule of M, then $N \oplus K$ is an essential submodule of M (where K is the relative intersection complement.) Since M is non-singular, then by [10] $N \oplus K$ is dense submodule of M. Thus by hypothesis $N \oplus K$ isafinitely pseudo-injective submodule of M. Hence by [8] N is a finitely pseudo-injective submodule of M. Therefore M is FP-Module.■

Before we give the last result of this suction, we introduce the following lemma

Lemma 6.6 [15, Th. 4.3]

For any finitely pseudo-injective module, if $S = End_R(M)$, then $J(S) = \{\alpha \in S: Ker\alpha\ is\ essential\ in\ M\}$

Theorem 6.7

Let M be an R-module such that $J(End_R(M)) = (0)$ then M is FP-Module if and only if M is afinitely pseudo-injective and every quasi-invertible submodule of M is a finitely pseudo-injective.

Proof

$(\Rightarrow)$ Trivial.

$(\Leftarrow)$ Let N be a submodule of M, then $N \oplus K$ is essential submodule of M (where K is the relative intersection complement of N). We claim that $N \oplus K$ is a quasi-invertible submodule of M. Let $g \in Hom_R(M/N \oplus K, M)\ and\ g \neq 0.$ Define $f = g \circ \pi$ where $\pi: M \to M/N \oplus K$ a natural homomorphism is. Hence $f \in End_R(M)$ and $f \neq 0\ and\ N \oplus K \subseteq kerf.$ Since $N \oplus K$ is an essential submodule of M, then $Kerf$ is essential submodule of M. Since M is a finitely pseudo-injective, then $f \in J(End_R(M))$ and $f = 0$, this implies that $g = 0$, this is a contradiction. Therefore $Hom_R(M/N \oplus K, M) = (0)$, and hence $N \oplus K$ is a quasi-invertible submodule of M. Thus by hypothesis $N \oplus K$ is a finitely pseudo-injective submodule of M. Hence by [8] N is afinitely pseudo-injective. Thus M is FP-Module.

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المقاسات من النمط FP ومفاهيم ذات علاقة

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الخلاصة:

في هذا البحث قدمنا تعميم لمفهوم المقاسات من النمط Q والتي أسميناها المقاسات من النمط FP.حيث أعطينا العديد من الأمثلة والخواص والتشخيصات للمقاسات من النمط FP. من جانب أخر بحثنا عن الشروط الكافية التي بتوافرها تصبح المقاسات من النمط FP مقاسات من النمط Q. فضلاً عن ذلك المقاسات التي لها علاقة بالمقاسات من النمط FP.