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Decomposition of Homeomorphisms on Intuitionistic Topological Spaces.

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ABSTRACT

The aim of this paper is to introduce two new classes of generalized homeomorphisms on intuitionistic topological spaces and shown that one of these classes has a group structure. Moreover some properties of these two homeomorphisms are obtained.

INTRODUCTION

Generalization of the concept of closed set was given by Levine[5] . Bhattacharyya and Lahiri [2] have generalized the concept of closed sets to semi-generalized various topological properties are given. Arya and Nour [1] had defined generalized semi-open sets with help of semi openness and used them to obtain some characterizations of S-normal spaces. Devi, Balachandran and Maki [4].

Defined two classes of maps called semi-generalized homeomorphisms and generalized semi-homeomorphisms and also defined two classes of maps called sgc-homeomorphism and gsc-homeomorphism. Ahmed Ozcelik and Serkan Narli [7].

Studied two classes of generalized homeomorphism and some properties of them., Yaseen in[9,10,11] studied a generalized closed set in intuitionistic topological spaces. Raouf [8].

Generalized closed set and studied several generalizations of homeomorphism between intuitionistic topological spaces in intuitionistic topological spaces. In this paper we introduce two classes of maps between intuitionistic topological spaces (ITS) called sgs-hmoeomrphisms and gsg-homeomorphisms and study their properties.

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Throughout the present paper, (X,T) and (Y,Ψ) denote an intuitionistic topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be an IS inX we denote the interior of A (respectively the closure of A with respect to an intuitionistic topological space)(ITS) T by $int(A)$ (respectively $cl(A))$.

PRELIMINARIES

Since we shall use the following definitions and some properties, we recall them in this section.

- A subset B of ITS (X,T) is said to be semi-closed if there exists a closed set F such that $int(F) \subseteq B \subseteq F$. A subset B of (X,T) is called a semi-open set, its complement $\bar{B}$ is semi-closed in (X,T). Every closed (respectively open) set is semi-closed (respectively semi-open)[10].
- Let X be a non-empty set, and let A and B are IS, having the form $A = \langle x, A_1, A_2 \rangle$; $B = \langle x, B_1, B_2 \rangle$ respectively. Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of IS in X, where $A_i = \langle x, A_i^{(1)}, A_i^{(2)} \rangle$, then,
- 1) $\tilde{\emptyset} = \langle x, \emptyset, X \rangle$; $\tilde{X} = \langle x, X, \emptyset \rangle$.
  2) $A \subseteq B, iff\ A_1 \subseteq B_1\ and\ A_2 \supseteq B_2$.

- 3) the complement of A is denoted by $\bar{A}$ and defined by $\bar{A} = \langle x, A_2, A_1 \rangle$.
- 4) $\cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle; \cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle$
- Let X, Y be non-empty sets and let $f: X \to Y$ be a function. a) If $B = \langle y, B_1, B_2 \rangle$ is IS in Y, then the preimage of B under f is denoted by $f^{-1}(B)$ is IS in X defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$.
- b)If $A = \langle x, A_1, A_2 \rangle$ is an IS in X ,then the image of A under f is defined $by\ f(A) = \langle y, f(A_1), \underline{f}(A_2) \rangle, where \underline{f}(A_2) = (f(A_2^c))^c$
- A map $f: (X,T) \to (Y,\Psi)$ is said to be semi-closed if the image $f(F)$ of each closed F in (X,T) is semi-closed in (Y,Ψ). Every closed mapping is semi-closed [10,11]
- An intuitionistic topology (IT, for short), on a non-empty set X, is a family T of an IS in X containing $\tilde{\emptyset}, \tilde{X}$ and closed under arbitrary unions and finitely intersections. The pair (X,T) is called an Intuitionistic topological space (ITS, for short).
- Let (X,T) be ITS and A be a subset of X. then, the semi-interior and semi-closure of A are defined by:
- $Sint(A) = \cup \{G_i : G_i\ is\ semi-open\ in\ X\ and\ G_i \subseteq A\}$
- $Scl(A) = \cap \{K_i : K_i\ is\ semi-closed\ in\ X\ and\ A \subseteq K_i\}$ [9].
- A subset B of IT$S(X,T)$is said to be semi generalized-closed (sg-closed) if $Scl(B) \subseteq U$ when ever $B \subseteq U$ and U is semi-open. The complement of a sg-closed set is called sg-open. Every semi-closed set is sg-closed. The concepts of g-closed sets and sg-closed set are in general, independent. The family of all sg-closed set of any ITS (X,T) is denoted by sgc(X)[8].
- A subset B of ITS (X,T) is said to be generalized semi-open(gs-open) if $F \subseteq Sint(B)$ whenever $F \subseteq B$ and F is closed. B is generalized semi-closed (gs-closed) if and only if $\bar{B}$ is gs-open. Every closed set (semi-closed set, g-closed set and sg-closed set) is gs-closed. The family of all gs-closed set of any ITS (X,T) is denoted by gsc(X).[8]
- A map $f: (X,T) \to (Y,\Psi)$ is called a semi generalized-continuous (sg-continuous mapping) if $f^{-1}(V)$ is sg-closed in X for every closed set V of Y.[8].
- A map $f: (X,T) \to (Y,\Psi)$ is called a generalized semi-continuous (gs-continuous mapping) if $f^{-1}(V)$ is gs-closed in X for every closed set V of Y.[8].
- A map $f: (X,T) \to (Y,\Psi)$ is called semi-generalized closed map (respectively semi-generalized open map) if $f(V)$ is semi generalized-closed (respectively semi-generalized open) in Y for every closed (respectively, open) set V in X. Every semi-closed map is semi generalized–closed. A semi generalized-closed map (resp. semi generalized-open map) is written as sg-closed map (resp. sg-open map).[8]
- A map $f: (X,T) \to (Y,\Psi)$ is called generalized semi- closed map (respectively generalized semi-open map) if $f(V)$ is gs- closed (respective gs-open) in Y for every closed (respectively, open) set V in X. Every semi-closed map is generalized semi–closed. A generalized semi-closed map (resp. generalized semi-open map) is written as gs-closed map (resp. gs-open map).[8]
- A map $f: (X,T) \to (Y,\Psi)$ is said to be a semi homeomorphism (B) (simply s.h.B) if f is continuous, f is semi-open(i.e. f(U) is semi-open for every open set U of X) and f is bijective[11]
- A map $f: (X,T) \to (Y,\Psi)$ is said to be semi-homeomorphism (C.H.) (simply s.h.(C.H)) if f is irresolute(i.e.$f^{-1}(V)$ is semi-open for every semi-open set V of Y), f is pre-semi-open(i.e.f(U) is semi-open for every semi-open set U of X)and f is bijective [8].
- A map $f: (X,T) \to (Y,\Psi)$ is said to be sg-irresolute map if .$f^{-1}(V)$ is sg-closed set in for every sg-closed set V of Y[8] .

- A map $f:(X,T)\rightarrow(Y,\Psi)$ is said to be gs-irresolute map if $f^{-1}(V)$ is gs-closed set in for every gs-closed set V of Y [8].
- A bijection $f:(X,T)\rightarrow(Y,\Psi)$ is called a semi-generalized homeomorphism (sg-homeomorphism) if f is both sg-continuous and sg-open [8].
- A bijection $f:(X,T)\rightarrow(Y,\Psi)$ is called a g-homeomorphism if f is g-continuous and g-closed [8].
- A bijection $f:(X,T)\rightarrow(Y,\Psi)$ is called a gc-homeomorphism if f is g-irresolute and its inverse f^{-1} is also g-irresolute [8].
- .A bijection $f:(X,T)\rightarrow(Y,\Psi)$ is called a sgc-homeomorphism if f is sg-irresolute and its inverse f^{-1} is also sg-irresolute [8] .
- A bijection $f:(X,T)\rightarrow(Y,\Psi)$ is called a generalized semi-homeomorphism (gs-homeomorphism) if f is both gs-continuous and gs-open [8].
- A bijection $f:(X,T)\rightarrow(Y,\Psi)$ is called a gsc-homeomorphism if f is gs-irresolute and its inverse f^{-1} is also gs-irresolute [8].
- An ITS space (X,T) is called $T_{1/2}$ if every g-closed set is closed, that is ifand only if every gs-closed set is semi-closed [4].
- An ITS (X,T) is called a T_b space if every gs-closed set is closed [4].

Remark 2.1

- The notions g-closed set and sg-closed set in ITS are independent notions. The following examples show the cases.

Example2.2 Let $X=\{a,b,c\};T=\{\tilde{\phi},\tilde{X},A,B\}$,*where* A$=\langle x,\{a\},\{b\}\rangle,\ B=\langle x,\{a,b\},\phi\rangle$.
$SOX=T\cup\{C,F,G\}$,where C$=\langle x,\{a\},\emptyset\rangle,\ F=\langle x,\{a,c\},\phi\rangle, G=\langle x,\{a,c\},\{b\}\rangle$.F is g-clsed set in X, but F is not sg-closed.

Example 2.3

Let $X=\{a,b,c\};T=\{\tilde{\phi},\tilde{X},A,B,C\}$,where A$=\langle x,\{c\},\{a,b\}\rangle,\ B=\langle x,\{b\},\{c\}\rangle\ and\ C=\langle x,\{b,c\},\emptyset\rangle$.
$SOX=T\cup\{F,M\}$, where F$=\langle x,\{c\},\{b\}\rangle,\ M=\langle x,\{a,b\},\{c\}\rangle$.B is sg-clsed set in X, but B is not g-closed.

GSHOMEOMORPHISM

In this section, the relation between semi-homeomorphism (B) and gsc-homeomorphism is investigated and the diagram of implications is given. Also the gsg-homeomorphism is defined and some of its properties are obtained.

Remark3.1

The following two examples show that the concept of

1. Semi-homeomorphism (B) and gsc-homeomorphism are independent of each other.
2. Semi-homeomorphism (C.H) and gsc-homeomorphism are independent.

We start with example showing that:

1. gsc-homeomorphism,but not semi-homeomorphism (B) and not homeomorphism (C.H).
2. gsg- homeomorphism,but not g-homeomorphism and no gc-homeomorphism.
3. gsg- homeomorphism,but not semi-homeomorphism (B) and not homeomorphism (C.H).

Example 3.2

Let $X=\{1,2,3\};T=\{\tilde{\phi},\tilde{X},A,B,C\}$,where A$=\langle x,\{2\},\{1,3\}\rangle,\ B=\langle x,\{1\},\{2\}\rangle\ and\ C=\langle x,\{1,2\},\emptyset\rangle$ .Let $Y=\{a,b,c\};T=\{\tilde{\phi},\tilde{Y},D,E,F\}$,where D =<y,{a},{c}>,E=<y,{c},{a,b}> and F=<y,{a,c},$\emptyset$>. Define a mapping $f:X\rightarrow Y\ by\ f(2)=a, f(3)=b\ and\ f(1)=c$.
$SOX=T\cup\{\bar{H},V\}$
,where $V=\langle x,\{1,3,\{2\}\}\rangle,\bar{H}=\langle x,\{2\},\{1\}\rangle$.
$SOY=\Psi\cup\{\gamma_6,\gamma_9\}$, where
$\gamma_6=\langle y,\{c\},\{a\}\rangle,\gamma_9=\langle y,\{a,b\},\{c\}\rangle$.
$gscY=\{\tilde{\emptyset},\tilde{Y},D,E,\gamma_3,\gamma_6,\gamma_8,\gamma_9,\gamma_{10},\gamma_{11}\}$, where
$\gamma_3=\langle y,\{a\},\{b,c\}\rangle,\gamma_8=\langle y,\{a,b\},\emptyset\rangle,\gamma_{10}=\langle y,\{b,c\},\emptyset\rangle,\gamma_{11}=\langle y,\{b,c\},\{a\}\rangle$.

$gsc(X) = \{\tilde{\phi}, \tilde{X}, \bar{H}, M, A, B, O, I, W, V\}$, where M$= \langle x, \{1\}, \{2,3\}\rangle$, $O = \langle x, \{2,3\}, \emptyset\rangle$ and $I = \langle x, \{2,3\}, \{1\}\rangle$ and W$= \langle x, \{1,3\}, \emptyset\rangle$. Therefore we can see that (1)- f is gsc-homeomorphism, but f is not semi-homeomorphism(B)and not semi-homeomorphism(C.H), and (2)- f is gsg-homeomorphism, but not homeomorphism (g-homeomorphism, gc-homeomorphism, semi-homeomorphism (B) and not homeomorphism (C.H)).

The next example shows that there is a semi-homeomorphism(B), but not gsc-homeomorphism.

Example 3.3

Let $X = \{a, b, c\}; T = \{\tilde{\phi}, \tilde{X}, A, B\}$, where A$= \langle x, \{b\}, \{a, c\}\rangle and\ B = \langle x, \{b\}, \{a\}\rangle$ Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, D\}$, where D=<y,{1},{2}>.

Define a mapping $f: X \to Y\ by\ f(a) = 2, f(b) = 3 and\ f(c) = 1$ .we can see that f is semi-homeomorphism(B).

$SOY = \Psi \cup \{\alpha_1, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\}$, where $\alpha_1 = \langle y, \{3\}, \emptyset\rangle, \alpha_3 = \langle y, \{3\}, \{2\}\rangle, \alpha_4 = \langle y, \{1,3\}, \emptyset\rangle, \alpha_5 = \langle y, \{1,3\}, \{2\}\rangle, \alpha_6 = \langle y, \{2,3\}, \emptyset\rangle, \alpha_7 = \langle y, \{2,3\}, \{1\}\rangle$

. We can see also that f is not gsc-homeomorphism., and f is sg-homeomorphism, but not gsg-homeomorphism.

The next example shows that f is semi-homeomorphism(C.H), but not gsc-homeomorphism

Example 3.4

Let $X = \{a, b, c\}; T = \{\tilde{\phi}, \tilde{X}, A, B\}$, where A$= \langle x, \{a\}, \{b\}\rangle and\ B = \langle x, \{a, b\}, \emptyset\rangle$ Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, C\}$, where C=<y,{1},{2}>.

Define a mapping $f: X \to Y\ by\ f(a) = 1, f(b) = 2 and\ f(c) = 3$.

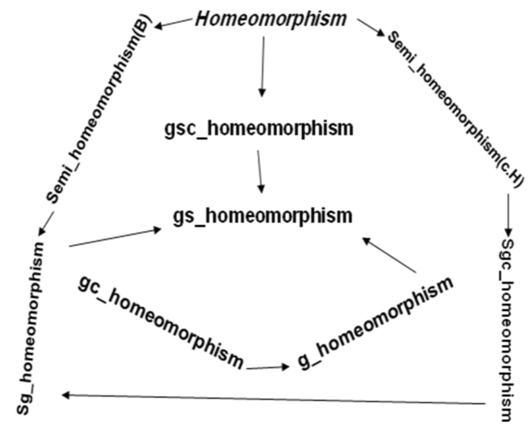
$SOX = T \cup \{Q_1, Q_4, Q_5\}$ Where $Q_1 = \langle x, \{a\}, \emptyset\rangle, Q_4 = \langle x, \{a, c\}, \emptyset\rangle, Q_5 = \langle x, \{a, c\}, \{b\}\rangle$.

$SOY = \Psi \cup \{D, G, I, J\}$, where G$= \langle y, \{1,2\}, \emptyset\rangle, I = \langle y, \{1,3\}, \emptyset\rangle, J = \langle y, \{1,3\}, \{2\}\rangle and\ D = \langle y, \{1\}, \emptyset\rangle$ we can see that:(a) f is semi-homeomorphism(C.H), But f is not gsc-homeomorphism and not gsg-homeomorphism. (b) f is sgs-homeomorphism, but not gsc-homeomorphism.

The following result gives the relation among different type of homeomorphism defined above.

Proposition 3.5

The following implications are valid while the reverse implications are not.



In the following example we show:

There is f which is g-homeomorphism, but not gc-homeomorphism.

There is f which is gs-homeomorphism, but not gsc-homeomorphism.

There is f which is sg-homeomorphism, but not sgc-homeomorphism.

Example 3.6

Let $X = \{a, b, c\}; T = \{\tilde{\phi}, \tilde{X}, A, B, C\}$, where A$= \langle x, \{c\}, \{a, b\}\rangle and\ B = \langle x, \{a\}, \{b, c\}\rangle\ and\ C = \langle x, \{a, c\}, \{b\}\rangle$ Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, D, E\}$, where D$= \langle y, \{2\}, \{3\}\rangle\ and\ E = \langle y, \emptyset, \{2,3\}\rangle$. Define a mapping $f: X \to Y\ by\ f(a) = 1, f(b) = 3 and\ f(c) = 2$.

$SOY = \Psi \cup \{P_i\}_{i=1}^{3}$, where $P_1 = \langle y, \{2\}, \emptyset\rangle,\ P_2 = \langle y, \{2,3\}, \emptyset\rangle\ and\ P_3 = \bar{P_1}$.

$SOX = T \cup \{G, K, L, N, W\}$, where G $= \langle x, \{c\}, \{a\}\rangle, K = \bar{G},\ L = \langle x, \{a, c\}, \emptyset\rangle, N = \langle x, \{a, b\}, \{c\}\rangle$, and W$= \langle x, \{b, c\}, \{a\}\rangle$ .We can see that:

f is g-homeomorphism, but not gc-homeomorphism.

f is gs-homeomorphism, but not gsc-homeomorphism,

f is sg-homeomorphism, but not sgc-homeomorphism.
f is gs-homeomorphism, but not sgs-homeomorphism.
f is sg-homeomorphism, but not sgs-homeomorphism.

Example 3.7

a. Recall Example 3.2 we can get the following:
b. f is gs-hom, but not g-hom;
c. f is gsc-hom, but not gc-hom and not hom.
d. f is sg-hom, but not s.h.(B);
e. f is sgc-hom, but not s.h.(C.H);
f. f is sgc-hom., but not s.h.(B).

Example 3.8

Let $X = \{a, b, c\}; T = \{\tilde{\phi}, \tilde{X}, A, B\}$, where A $= \langle x, \{c\}, \{b\}\rangle and\ \ B = \langle x, \{a, c\}, \emptyset\rangle$ Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, C, D\}$, where C=<y,{1,2}, Ø> and D=<y, {1},{3}>. Define a mapping $f: X \to Y\ by\ f(a) = 1, f(b) = 3 and\ f(c) = 2$.
$SOX = T \cup \{E, H, I\}\ Where\ E = \langle x, \{c\}, \emptyset\rangle, H = \langle x, \{a, c\}, \{b\}\rangle, I = \langle x, \{b, c\}, \emptyset\rangle$. we can see :f is gs-hom., but not sg-hom.

Example 3.9

Recall Example 3.3. It is clear that f is s.h.(B), but not homeomorphism.
Recall Example 3.4 .we see that f is s.h.(C.H), but f is not homeomorphism.

Remark 3.10

a. gsc-homeomorphism and gc-homeomorphism are independent.
b. sgc-homeomorphism and s.h.(B) are independent.

Note that Example 3.2 show the case : f is gsc-homeomorphism, but f is not semi-homeomorphism(B)and not semi-homeomorphism(C.H).

Example 3.7 (b) show that f is gsc-hom, but not gc-hom and not hom.

Example 3.11

Let $X = \{a, b\}; T = \{\tilde{\phi}, \tilde{X}, A\}$, where A $= \langle x, \{a\}, \{b\}\rangle$ Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, C, D\}$, where C=<y,{2},{3}> D=<y, Ø ,{2,3}>. Define a mapping $f: X \to Y\ by\ f(a) = 2 and f(b) = 3$.
$SOX = T \cup \{E\}\ Where\ E = \langle x, \{a\}, \emptyset\rangle$,.
$SOY = \Psi \cup \{H, G, J\}$, where G $= \langle y, \{2\}, \emptyset\rangle, H = \langle y, \{2,3\}, \emptyset\rangle$, and $J = \langle y, \emptyset, \{2\}\rangle$
we can see that: 1) f is semi-homeomorphism(B), But f is not sgc-homeomorphism and not gsg-homeomorphism. 2) f is homeomorphism, but not gsg-homeomorphism. 3)f is g-homeomorphism and gc-homeomorphism, but not gsg-homeomorphism.

Remark 3.12

The concepts of s.h.(B) and s.h.(CH) are independent.
The following examples show the cases.

Example 3.13

We show in this example that f is s.h.(C.H), but not s.h.(B).
Let $X = \{a, b, c\}; T = \{\tilde{\phi}, \tilde{X}, A, B\}$, where A $= \langle x, \{b\}, \{a, c\}\rangle and\ \ B = \langle x, \{b\}, \emptyset\rangle$ Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, D, E\}$, where D=<y,{2},{1,3}> and E=<y,{1,2},{3}>. Define a mapping
$f: X \to Y\ by\ f(a) = 1, f(b) = 2 and\ f(c) = 3$.
$SOX = T \cup \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}\ Where\ Q_1 = \langle x, \{b\}, \{a\}\rangle, Q_2 = \langle x, \{b\}, \{c\}\rangle, Q_3 = \langle x, \{a, b\}, \{\emptyset\rangle, Q_4 = \langle x, \{a, b\}, \{c\}\rangle, Q_5 = \langle x, \{b, c\}, \{\emptyset\rangle and\ Q_6 = \langle x, \{b, c\}, \{a\}\rangle$.
$SOY = \Psi \cup \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$, where
$\alpha_1 = \langle y, \{2\}, \emptyset\rangle, \alpha_2 = \langle y, \{2\}, \{1\}\rangle, \alpha_3 = \langle y, \{2\}, \{3\}\rangle, \alpha_4 = \langle y, \{1,2\}, \emptyset\rangle, \alpha_5 = \langle y, \{2,3\}, \emptyset\rangle, \alpha_6 = \langle y, \{2,3\}, \{1\}\rangle$ we can see that f is semi-homeomorphism(C.H). But f is not semi-homeomorphism (B).

Example3.14

Let $X = \{a, b, c\}; T = \{\tilde{\phi}, \tilde{X}, A, B, C\}$, where A $= \langle x, \{c\}, \{a, b\}\rangle\ B = \langle x, \{b\}, \{c\}\rangle\ \ and\ \ C = \langle x, \{b, c\}, \emptyset\rangle$ Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, D\}$, where D=<y,{2},{3}>.
Define a mapping $f: X \to Y\ by\ f(a) = 1, f(b) = 2 and\ f(c) = 3$.
$SOX = T \cup \{Q_1, Q_8\}\ Where\ Q_1 = \langle x, \{c\}, \{b\}\rangle, Q_8 = \langle x, \{a, b\}, \{c\}\rangle$.
$SOY = \Psi \cup \{\beta_1, \beta_2, \beta_3, \beta_4\}$,, where
β1=<y,{2}, Ø>,β2=<y,{2,3}, Ø>, β 3 =<y,{1,2}, Ø> and β4 =<y,{1,2},{3}> we can see that f is semi-

homeomorphism(B). But f is not semi-homeomorphism (C.H).

Next we are going to define the gsg-irresolute and gsg-homeomorphism.

Definition3.15

A mapping $f:(X,T)\rightarrow(Y,\Psi)$, where $(X,T)\ and (Y,\Psi)$ are ITS is called gsg-irresolute map if the inverse image of every gs-closed set in Y is sg-closed set in X.

Definition3.16

A bijective mapping $f:(X,T)\rightarrow(Y,\Psi)$, where $(X,T)\ and (Y,\Psi)$ are ITS is called gsg-homeomorphism, if both f and its inverse are gsg-irresolute. If there exists a gsg-homeomorphism between $(X,T)\ and (Y,\Psi)$, then (X,T) and (Y,Ψ) are said to be gsg-homeomorphicm. The family of all gsg-homeomorphicm ITS to ITS (X,T) is denoted by gsgh(X).

Note : Example 3.11, 3.4 and 3.2 shows that gsg-homeomorphism and homeomorphism (sh(C.H),sh(B),g-hom. and gc-hom.) are independent.

Remark 3.17

Every gsg-homeomorphism implies both gsc-homeomorphism and sgc-homeomorphism.

Proof: Since f is gsg-homeomorphicm.So f is gsg-irresolute. Let V be any gs-closed in Y, then $f^{-1}(V)$ is sg-closed in X. Since f is gsg-irresolute then $f^{-1}(V)$ is sg-closed for every sg-closed in Y. Also since f is gsg-homeomorphism,then f^{-1} is gsg-irresolute (by definition). Let V be gs-closed set in X, $(f^{-1})^{-1}(V)=f(V)$ is sg-closed in X. So f(V) is sg-closed for every sg-closed in Y.Thus f, f^{-1} is sg-irresolute. Therefore f is sgc-homeomorphism. It is clear that f is gsg-homeomorphism, then f is gsc-homeomorphism.

However the converse is not true as shown by the following example.

Example 3.18

Let $X=\{a,b,c\}; T=\{\tilde{\phi},\tilde{X},A,B,C\}$, where A$=\langle x,\{a,b\},\{c\}\rangle, B=\langle x,\{a,c\},\emptyset\rangle and$ $C=\langle x,\{a\},\{c\}\rangle$ Let $Y=\{1,2,3\}; \Psi=\{\tilde{\phi},\tilde{Y},D,E,F\}$, where D=<y,{1,2},{3}>, E=<y,{1,3},$\emptyset$> and F=<y,{1},{3}>. Define a mapping $f:X\rightarrow Y\ by\ f(a)=2, f(b)=1 and\ f(c)=3$.

$SOX=T\cup\{Q_1,Q_2\}\ Where\ Q_1=\langle x,\{a,b\},\emptyset\rangle and Q_2=\langle x,\{a\},\emptyset\rangle$,

$SOY=\Psi\cup\{\beta_1,\beta_2\}$, where $\beta_1=\langle y,\{1,2\},\emptyset\rangle, and \beta_2=\langle y,\{1\},\emptyset\rangle$. We can see that f is both gsc-homeomorphism and sgs-homeomorphism . Since gsc(X)=$\{\tilde{\phi},\tilde{X},Q_1\}$, $sgc(X)=\{\tilde{\phi},\tilde{X}\}$ and sgc of (Y)=$\{\tilde{\phi},\tilde{Y},\beta_1\}$ and sgc(Y)=$\{\tilde{\phi},\tilde{Y}\}$so f is not gsg-homeomorphism.

The following proposition is a direct consequence of definitions.

Proposition 3.19

Every gsg-homeomorphism implies both gs-homeomorphism and sg-homeomorphism. Example 3.3 shows that the converse of (Prop.3.19) is not true in general.

Next we are going to generalize the definition of cardinal number for IS

Definition 3.20

Let X be any set and $A=\langle x,A_1,A_2\rangle$ be IS in X. then the cardinal number of A is denoted by #A defined as follows: $\#A=(\alpha,\beta)$ where $\alpha=\#A_1,\beta=\#A_2$.

Remark

From definition 3.20 we can get the following properties of cardinal number of IS.

a. If $\#A=(\alpha,\beta)$ and $\#B=(\gamma,\delta)$ then. $\#A=\#B\ if\ and\ only\ if \alpha=\gamma\ and\ \beta=\delta$.
b. $\#(A\cup B)=(\alpha+\gamma,\beta+\delta)$, whenever $A\cap B=\tilde{\emptyset}$.
c. $\#(\tilde{\emptyset})=(0,\#X) and\ \#\tilde{X}=(\#X,0)$;

d. $\#A + \#B = (\alpha+\gamma, \beta+\delta)$ and $\#A \cdot \#B = (\alpha\gamma, \beta\delta)$

Definition 3.21

Let (X,T) and (Y,Ψ) be ITS. If the following properties are satisfied;

a. $sgc(X) = gsc(X)$ and $sgc(Y) = gsc(Y)$;

There exists a bijective map $\vartheta: gsc(X) \to gsc(Y)$ such that for each $A \in gsc(X), \#\vartheta(A) = \#(A)$ .then the space (X,T) and (Y,Ψ) are called S-related.

The following theorem follows from definition of gsg-homeomorpsm and (Def.3.21).

Theorem 3.22

The space (X,T) and (Y,Ψ) are gsg-homeomorphic if and only if these spaces are S-related.

Theorem 3.23

a. Every gsc(sgc) homeomorphism from $T_{1/2}$ space onto itself is a gsg-homeomorphism.

b. Every gs(sg) homeomorphism from T_b space onto itself is gsg-homeomorphism.

Proof:

c. Since for any $T_{1/2}$ (X,T) the family of sg-closed set is equal to the family of gs-closed sets. Any gsc (sgc)-homeomorphism from X onto X is gsg-homemorphism. In any T_b (X,T) every gs-closed subset is a closed subset so (b) is obvious.

d. As direct consequences of theorem 3.23, we have the following corollary.

Corollary 3.24

Let (X,T) and (Y,Ψ) be any ITS's. if there exist any gsg-homeomorphism from X to Y, then every gsc(sgc)-homeomorphism from X to Y is sgc(gsc)-homeomorphism.

The following theorem gives relations among gsg, gsc, gs, sgc and sg-homeomorphism

Theorem 3.25

Let (X,T) be ITS ,then the following inclusions holds.

a. $gsgh(X) \subseteq gsch(X) \subseteq gsh(X)$ and

b. $gsgh(X) \subseteq sgch(X) \subseteq sgh(X)$.

c. If gsgh(X) ≠ $\emptyset$,then gsgh(X) is agruop and sgch(X) =gsch(X)=gsgh(X).

Proof:

It follows from Remark 3.17 and Cor.3.24.

Theorem 3.26

If $f: (X,T) \to (Y,\Psi)$ is gsg −homeomorphism, then it induces an isomorphism from the group $gsgh(X)$ onto the group $gsgh(Y)$.

Proof: *The homomorphism $f_*: gsgh(X) \to gsgh(Y)$ is induced from f by $f_*(h) = f \circ h \circ f^{-1}\ \forall h \in gsgh(X)$. Then it easily follows that f_* is an isomorphism.*

SGS-HOMEOMRPHISM

In this section we introduce a new kind of homeomorphism namely sgs-homeomorphism and related to other kind of homeomorphism which are defined in this work

Definition 4.1

A map $f: (X,T) \to (Y,\Psi)$ is called a sgs-irresolute map if the inverse image of each sg-closed set in Y is gs-closed in X.

Definition 4.2

A bijective map $f: (X,T) \to (Y,\Psi)$ is called a sgs-homeomorphism if f and its inverse are both sgs-irresolute maps.

If there exists a sgs-homeomorphism from X to Y, then the spaces (X,T) and (Y,Ψ) are said to be sgs-homeomorphic spaces.

Remark 4.3

Every sgc-homeomorphism andgsc-homeomorphism implises a sgs-homeomorphism. But the converse is not true in general see (Example 3.4), which is shows that sgs- homeomorphism is not gsc-homeomorphism.

The following example shows that sgs-homeomorphism is not sgc-homeomorphism and not sg-homeomorphism.

Example 4.4

Let $X = \{a,b,c\}; T = \{\tilde{\phi}, \tilde{X}, A\}$,where A $= \langle x, \{c\}, \{b\} \rangle$, .Let $Y = \{1,2,3\}; \Psi = \{\tilde{\phi}, \tilde{Y}, B, C\}$,where B=<y,{1},{2,3}>, C=<y,{2,3},{1}>. Define a

mapping
$f: X \to Y$ by $f(a) = 2, f(b) = 1$ and $f(c) = 3$.

$SOX = T \cup \{\alpha_1, \alpha_2, .., \alpha_7\}$ Where $\alpha_1 = \langle x, \{c\}, \emptyset \rangle$ and $\alpha_2 = \langle x, \{c\}, \{a\} \rangle, \alpha_3 = \langle x, \{c\}, \{b\} \rangle$ and $\alpha_4 = \langle x, \{a,c\}, \emptyset \rangle, \alpha_5 = \langle x, \{a,c\}, \{b\} \rangle$ and $\alpha_6 = \langle x, \{b,c\}, \emptyset \rangle$ and $\alpha_7 = \langle x, \{b,c\}, \{a\} \rangle$. $SOY = \Psi$,

where . We can see that f is sgs-homeomorphism but not sgc-homeomorphis , sg-homeomorphism and f is not homeomorphism. Also f sgs-homeomorphism, but f is not gsg-homeomorphism.

Remark 4.5

Every homeomorphism is a sgs-homeomorphism.
Proof: : Since f is homeomorphicm.So f is continuous. Let V be any closed set in Y, then $f^{-1}(V)$ is gs-closed set in X. Since f is continuous. So by every closed set is gs-closed set **and sg-closed set) ,then** $f^{-1}(V)$ **is gs-closed for** every sg-closed in Y. Thus f is sgs-irresolute. Also f^{-1} is continuous. Let V be a closed set, then $(f^{-1})^{-1}(V) = f(V)$ is gs-closed in X for every sg-closed in Y.Thus f, f^{-1} is sgs-irresolute. Therefore f is sgs-homeomorphism. But the converse is not true. See example 4.4.

Remark 4.6

Every sgs-homeomorphism is a gs-homeomorphism.
Proof: : Since f is sgs- homeomorphicm.So f is sgs-irresolute. Let V be any sg-closed set in Y, then $f^{-1}(V)$ is gs-closed set in X. Since f is sgs-irresolute. So by every closed set is sg-closed set ,then $f^{-1}(V)$ is gs-closed in X for every closed in Y. Thus f is gs-continuous and also f^{-1} is sgs-irresolute. Let V be a sg-closed set, then $(f^{-1})^{-1}(V) = f(V)$ is gs-closed in X for every closed in Y.Thus f^{-1} is gs-closed. Therefore f is gs-homeomorphism. But the converse is not true. See example 3.6.

Example 4.7

The mapping $f: (X, T) \to (Y, \Psi)$ in example 3.6 is gs-homeomorphism, but note sgs-homeomorphism.

Result 4.8

a. From example 4.7 we see that any sg-homeomorphism is not a sgs-homeomorphism.
b. Every gsg-homeomorphism is sgs-homeomorphism, but the converse is not true. Example 4.4 shows the case.

Theorem 4.9

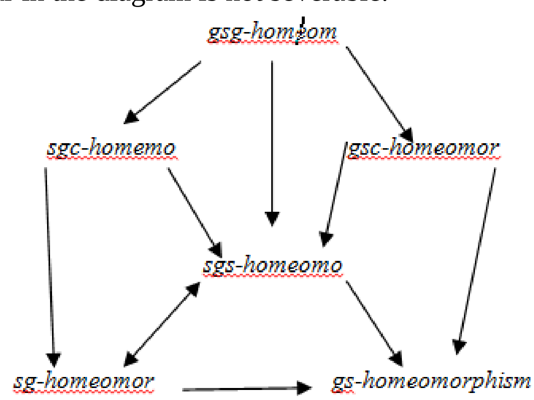
a. Every sgs-homeomorphism from a $T_{1/2}$ ITS onto itself is gsg-homemorphism. This implies that sgs-homeomorphism is both gsc-homeomorphism and sgc-homeomorphism.
b. Every sgs-homeomorphism from T_b ITS onto itself is a homeomorphism. This implies that sgs-homeomorphism is gs-homeomorphism, sg-homeomorphism, sgc-homeomorphism, gsc-homeomorphism, and gsg-homeomorphism.
c. Every sgc-homeomorphism from $T_{1/2}$ ITS space onto itself is a s.h.(C.H).

Proof: In a $T_{1/2}$ITS; every gs-closed set is a semi-closed set.

d. In a T_bITS; every gs-closed set is closed set.
e. Follows from definition of $T_{1/2}$ space.

CONCLUSION

In this paper, we introduce two classes of maps called sgs-homeomorphisms and gsg-homeomorphisms and study their properties. From all of the above statements, we have the following diagram, but the implication appear in the diagram is not severable.



Summary:

*Example 3.6 shows that **;***

1. *f is sg-homeomorphism ,but not sgc-homeomorphism and not sgs-homeomorphism;*
2. *f is gs-homeomorphism, but not gsc-homeomorphism and not sgs-homeomorphism;*

Example 3.4 shows that; f is sgs-homeomorphism, but not gsc-homeomorphism.
Example 3.8 show that; f is gs-homeomorphism, but not sg-homeomorphism
Example 3.18 shows that;

1. *f is sgs-homeomorphism, but not gsg-homeomorphism;*
2. *f is gsc-homeomorphism, but not gsg-homeomorphism.*

Example 4.4 shows that f is sgs-homeomorphism, but not sgc-homeomorphism, sg-homeomorphism and not gsg-homeomorphism.

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تركيب التشاكل التبولوجي على الفضاءات التبولوجية الحدسية

أسماء غصوب رؤوف

الخلاصة:

الهدف من هذا البحث هو تقديم نوعين جديدين من تعميمات تشاكل الهوميومورفزم في الفضاءات التبولوجية الحدسية وبينا ان احد هذين النوعين يملك تركيب زمرة . فضلاً عن ذلك حصلنا على صفات لهذين النوعين من التشاكل التبولوجي.