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On Weak Forms of Reqular Generalized Some Separation Axioms In Intuitionistic Topological Spaces

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ABSTRACT

Our goal in this paper is to give new definition of regular generalized T1 and regular generalized T2 separation axioms in intuitionistic topological spaces and study relations among several types of regular generalized separation axioms with some basic properties and counter examples.

Introduction:

The concept of "Intuitionistic fuzzy sets" was introduced by Atanassov in 1983[1] (IFS for short) , on the other hand Coker[4] introduced the notions of intuituinistic fuzzy First , we present the fundamental definitions.topological spaces .

In this paper , we introduced the concept of regular generalized T1 , locally regular generalized T2 separation axioms

Preliminaries:

First , we present the fundamental definitions. in intuitionsitic fuzzy topological spaces .We give some characterizations and basic properties for these concept .

Definitions 2.1[1].

Let X be anon empty fixed set-.An intuitionistic fuzzy set (IFS , for short)A is an object having the form A= < x , A1 , A2 > , which A1 and A2 are subset of X and satisfying the following $A_1 \cap A_2 = \phi$.

Definitions : 2.2[4].

An intuitionstic fuzzy topology (IFT. For short) on anonempty set X is afamily T containing $\tilde{\phi} =< x, \phi, X > and$ $\tilde{X} =< x, X, \phi >$ and closed under finite intersection and arbitrary union.

In this case the pair (X , T) is called an intuitionistic fuzzy topological spaces (IFTS, for short) and each IFS in T is known as an intuitionstic fuzzy open set (IFOS , for short) in X .

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The complement $\bar{A}$ of an IFOS A in an IFTS (X , T) is called an intuitionistic fuzzy closed set (IFCS , for short) , in X.

Definition : 2.3[4]

let X be anon empty set and let the IFS's A and B be in the form A=< x , A1 , A 2 > , B = < x , B1 , B2 > and let { Ai : i∈I } bean arbitrary family of IFS's in X .Then

i. A = B ⇔ A1 ⊆ B1 ∧ A2 ⊇ B2 ;
ii. A ⊆ B ⇔ A ⊆ B ∧ B ⊆ A ;
iii. $\bar{A}$ = < x , A2 , A1 >
iv. ∪Ai = < x , ∪A1 , ∩A2 > ; ∩Ai = < x , ∩A2 , ∪A2 >.

Definition : 2.4[4]

An intuitionstic fuzzy point in X (IP for short) is defined by $\tilde{p} =< x, \{p\}, \{p\}^c >$ and the IS $\tilde{\tilde{p}} =< x, \phi, \{p\}^c >$ in called avanishing intuitionstic point (VIP for short) in X.

Definition : 2.5[1]

Let A be an IFS , then the interior and closure of an IF'S A is defind by ;

Int

$$A = \cup\{G : G \in T, G \subseteq A\}$$

$$CLA = \cap\{k : \bar{k} \in T, A \subseteq k\}$$

Definition: 2.6

Let (X , T) be IFS , Asubset A of (X , T) is called regular generalized closed set (RgcS for short) if CLA $\subseteq$ A , whenever A $\subseteq$ U and U is regular open.

The complement of RgcS in X is called regular generalized open set (RgoS for short) in X.

Proposition : 2.7

Let (X , T) be IFS , A is RgoS in X if and only if for each regular closed set F such that F $\subseteq$ A , then F $\subseteq$ Int A.

Proof : $\Rightarrow$

Suppose that A is RgoS in X , then A is Rgc , so for each RoS in X and A ^{c} $\subseteq$ U , then CL A ^{c} $\subseteq$ U. Put A ^{c} = F and U $\subseteq$ Int A , then Int F $\subseteq$ U . $\therefore$ for each F $\subseteq$ A , F $\subseteq$ Int A.

$\Leftarrow$ suppose that A is RgcS in X then A ^{c} is Rgo , so for each F is RCS in X and F $\subseteq$ A ^{c} , F $\subseteq$ Int A ^{c}, so put CLF = U , then CLA $\subseteq$ U

$\therefore$ for each A $\subseteq$ U , CLA $\subseteq$ U.

Proposition : 2.8

If A is Rgc in IS'S (X , T) and $A \subseteq B \subseteq CLA$, then B is Rgc .

Proof :

Remark : 2.9

i)Intersection of any family of RgcS is Rgc.

ii)Any Union of RgoS is RgoS.

Proof : i

Let A , B be Two RgcS so for each U ROS in X , A $\subseteq$ U $\Rightarrow$ CL A $\subseteq$ U and for each V ROS , B $\subseteq$ V $\Rightarrow$ CLB $\subseteq$ V so A $\cap$ B $\subseteq$ U $\cap$ V , CL(A $\cap$ B) $\subseteq$ CLA $\cap$ CLB $\subseteq$ U $\cap$ V , So A $\cap$ B is RgcS in X.

ii is the duol of i

Remark :2.10

i)Every open set is RgoS but the converse is not true.

ii)Every closed set is RgcS , but the converse is not true.

Proof

Suppose that A is an open set , then for each RCS F $\subseteq$ A = Int A $\Rightarrow$ F $\subseteq$ Int $\Rightarrow$ A is Rgo and let A be closed set , so for each RgoS U , A $\subseteq$ U. CLA=A $\subseteq$ U $\Rightarrow$ A is Rgc.

Example : 2.11

Let X = { 1,2,3} and define T by T = { $\tilde{\phi}, \tilde{X}, A$ } weher A< X , {1} , {2,3} > so RC(x) = { $\tilde{\phi}, \tilde{X}$ } and Rgo (x) = { $\tilde{\phi}, \tilde{X}, A, B$ } where B=< X , {1,2} , {3} > , then B is Rgo but not open set and C = < X , {3} , {1,2} > is Rgc but no closed set.

For each U is ROS and B $\subseteq$ U we have prove that CL B $\subseteq$ U.

Suppose A is Rgc , then for each U is RO in (X , T) , A $\subseteq$ U then CL A $\subseteq$ U , but A $\subseteq$ B $\subseteq$ CLA so CLB $\subseteq$ CL(CLA) = CLA, then CLB $\subseteq$ CLA $\subseteq$ U i.e B is RgcS.

Proposition: 2.12

If A is an open and Rgc then A is closed set.

Proof:

Since A is open and Rgc , then $\forall U$ is Rgo in X , A $\subseteq$ U $\Rightarrow$ CLA $\subseteq$ U replacing U by A , we have A $\subseteq$ A

then CLA $\subseteq$ A ....(1)

but A $\subseteq$ CLA ....(2)

from (1) and (2) we have :

A=CLA i.e A is closed set.

3.The separation axiom Regular generalized T1:

In this section we introduce RGT separation axiom and study the basic properties and give this generalization with some details and conter examples.

Definition: 3.1

Let (X , T) be an ITS, (X,T) is said to be : -

a)RGT1 (i) if for each $x ,, y \in X, x \neq y$ there exists U,V where U,V are Rgo (X) s.t $\tilde{x} \in U, \tilde{y} \in V, \tilde{x} \neq V$, $\tilde{y} \notin U$

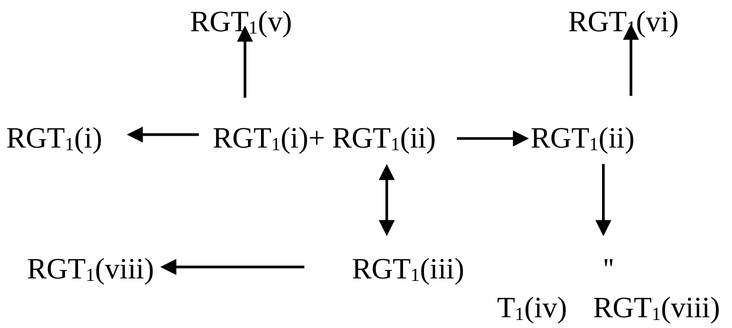
b)RGT1 (ii) if for each x,y $\in X, x \neq y$, there exists U,V where U,V are Rgo (X) s.t $\tilde{\tilde{X}} \in U, \tilde{\tilde{y}} \notin U$ and $\tilde{\tilde{y}} \in V$, $\tilde{\tilde{x}} \not\in \tilde{x} \in V$.

c)RGT1(iii) if for each $x, y \in X, x \neq y$, there exists U,V where U,V are Rgo (X) s.t. $\tilde{x} \in U \subseteq \bar{\tilde{y}}$ and $\tilde{y} \in V \subseteq \bar{\tilde{X}}$.

d)RGT1 (iv) if for each $x, y \in X, x, \neq y$ .there exists U,V where U,V are Rgo (X) s.t. $\tilde{\tilde{x}} \in U \subseteq \bar{\tilde{\tilde{y}}}$ and $\tilde{\tilde{y}} \in V \subseteq \bar{\tilde{\tilde{x}}}$.

e)RGT1(V) if for each $x, y \in X, x \neq y$, there exists U,V where U,V are Rgo(X) s.t $\tilde{y} \notin U$ and $\tilde{x} \notin V$.

f)RGT1 (vi) if for each $x, y \in X, x \neq y$ there exists U,V where U,V are Rgo (X) s.t $\tilde{\tilde{y}} \notin U$ and $\tilde{\tilde{x}} \notin V$.



Proof: RGT1(vi)⟶ RGT1(v)

Suppose $\forall x, y \in X, x \neq y$ so there exists U=< x,u1,u2> and V= <y,v1,v2> are Rgo(X) s.t $\tilde{x} =< x, \{x\}, \{x\}^c > \in U$ and $\tilde{y} =< y, \{y\}, \{y\}^c > \notin U$ and $\tilde{y} \in V, \tilde{x} \notin V$, this implies $\tilde{x} \notin V$ and $\tilde{y} \notin U$ there for RGT1(v) holds .

RGT1(ii) ⟶ RGT1(Vi)

Let $x, y \in X, x \neq y$, since RGT1(ii) hold so there exists U,V are Rgo(X) s.t $\tilde{\tilde{x}} \subset U, \tilde{\tilde{y}} \notin U$ and $\tilde{\tilde{y}} \in V$ where $\tilde{\tilde{y}} =< y, \phi, \{y\}^c >$ and

$\tilde{\tilde{x}} =< x, \phi, \{x\}^c > \notin \tilde{x} =< x, \{x\}, \{x\}^c > \in$V, from this we get $\tilde{\tilde{x}} \notin V$ and $\tilde{\tilde{y}} \notin U$ there for RGT1(vi) hold. RGT1(V) hold.

RGT1(i) + RGT1(ii)⇒RGT1(iii)

Let x,y∈X , $x \neq$y since RGT1(i)+RGT1(ii) holds so there exists $U =< x, U_1, U_2 >$ and V=<y,V1,V2> are Rgo(X) s.t $\tilde{x} \in U$ and $\tilde{y} \notin U, \tilde{y} \in V, x$ and $\tilde{\tilde{y}} \in V, \tilde{\tilde{x}} \notin \tilde{x} \in V$.

First we have to prove $\tilde{x} \in U \subseteq \tilde{\tilde{y}}$ and $\tilde{y} \in V \subseteq \tilde{\tilde{x}}$ we have to prove that U⊆$\tilde{\tilde{y}}$ and V⊆$\tilde{\tilde{x}}$ take U and $\tilde{\tilde{y}} =< y, \{y\}, \{y\} >$, since $\tilde{y} \notin U$ so $y \notin U$, there for $U_1 \subseteq \{y\}^c$ and $\{y\} \subseteq U_2$, this RGT1(i) + RGT1(ii)⇒RGT1(i) and RGT1(i) + RGT1(ii)⇒RGT1(ii) is direct RGT1(vi) ⇒ RGT1(v)

Suppose there exists U,V∈Rgo(X) s.t $\tilde{x} \in V, \tilde{y} \notin U$ and $\tilde{y} \in V, \tilde{x} \notin V$ this implies that $\tilde{x} \notin V$ and $\tilde{y} \notin U$ there for implies that $U \subseteq \tilde{\tilde{y}}$.

In similar way we can prove $V \subseteq \tilde{\tilde{x}}$ Hence RGT1(iii) halds

RGT1(iii) ⇒RGT1(i) + RGT1(ii)

We have to prove RGT1(iii) ⇒RGT1(i) and RGT1(iii) ⇒RGT1(ii) we prove that RGT1(iii) ⇒RGT1(i) , let x , y , $x \neq y$ .Since RGT1(iii) hold so there exists U,V ∈R.g.o(x)s.t $\tilde{x} \in U \subseteq \tilde{\tilde{y}}$ and $\tilde{y} \in V \subseteq \tilde{\tilde{x}}$ .Now $\tilde{x} \in U$, $\tilde{y} \notin$U,and $\tilde{y} \in$ V, $\tilde{x} \notin$V and $\tilde{y} \in \bar{U}$,so $\tilde{x} \in$U and $\tilde{y} \subseteq$V.Since $\tilde{y} \in$V⊆$\tilde{\tilde{x}}$ so RGT1 (i) holds.

We conuse similar argument to prove that RGT1(iii) ⇒RGT1(ii). RGT1(iii) ⇒RGT1(vii).

Suppose RGT1(iii) hold , take $x, y \in X$ s.t

RGT1(iii) ⇒RGT1(i) + RGT1(ii)

We have to prove RGT1(iii) ⇒RGT1(i) and RGT1(iii) ⇒RGT1(ii) we prove that RGT1(iii) ⇒RGT1(i) , let x , y , $x \neq y$ .Since RGT1(iii) hold

g)RGT1(vii) if for each $x \in X, \tilde{x}$ is Rgc(X).

h) RGT1(viii) if for each $x \in X, \tilde{\tilde{x}}$ is Rgc(X).

Theorem :3.2

Let (X , T) be an ITS , then the following implication are valied so there exists U,V ∈R.g.o(x)s.t $\tilde{x} \in U \subseteq \tilde{\tilde{y}}$ and $\tilde{y} \in V \subseteq \tilde{\tilde{x}}$ .Now $\tilde{x} \in U$, $\tilde{y} \notin$ U,and $\tilde{y} \in$ V, $\tilde{x} \notin$ V and $\tilde{y} \in \bar{U}$,so $\tilde{x} \in$ U and $\tilde{y} \subseteq$ V.Since $\tilde{y} \in$ V $\subseteq \tilde{\tilde{x}}$ so RGT1 (i) holds.

We conuse similar argument to prove that RGT1(iii) $\Rightarrow$RGT1(ii). RGT1(iii) $\Rightarrow$RGT1(vii). Suppose RGT1(iii) hold , take $x, y \in X$ s.t $x \neq y$ there exists U,V R.g.o(x) and $\tilde{x} \in U \subseteq \tilde{\tilde{y}}$ and $\tilde{y} \in V \subseteq \tilde{\tilde{x}}$ .since $\tilde{x} \in U$ so $x \in U$ we have to prove that $\tilde{x}$ is R.g.c , that is to prove is R.g.o (X) for if $\tilde{x} = U\{V : \tilde{y} \in V, V \in R.O(X)\}$,that is $\tilde{x}$ is union of ROS so it is R.g.o therefore $\tilde{x}$ is R.g.c .

RGT1(iv) $\Rightarrow$ RGT1(viii).

Suppose that RGT1(iv) hold and let $x \in X$ so for each $y \in X$ s.t $x \neq y$ there exists $U, V \in R.g.o(X)$ s.t $\tilde{\tilde{y}} \notin U$ and $\tilde{\tilde{x}} \notin V$.

We have to prove that $\tilde{x}$ is R.g.c that is we have to prove that $\tilde{\tilde{x}}$ is R.g,o(X) .

RGT1(iii) $\Rightarrow$RGT1(i) $\Rightarrow$RGT1(ii).

We have to prove RGT1(iii) $\Rightarrow$RGT1(i) and RGT1(iii) $\Rightarrow$RGT1(ii), First we prove that RGT1(iii) $\Rightarrow$RGT1(i), let $x, y \in X$, $x \neq y$ .Since RGT1(iii) hold so there exists R.g.o(X) s.t $\tilde{x} \in U \subseteq \tilde{\tilde{y}}$ and $\tilde{y} \in V \subseteq \tilde{\tilde{x}}$ .now $\tilde{x} \in V$, $\tilde{y} \notin U$ and $y \in U$, this implies that $\tilde{x} \in U$ and $\tilde{y} \in \bar{U}$, so in the same way we get that $y \in U$, $\tilde{x} \notin V$ for RGT1(i) holds .

We can use similar arqument to prove that RGT1(iii) $\Rightarrow$RGT1(ii) , RGT1(iii) $\Rightarrow$RGT1(vii) suppose RGT1(iii) hold let $x \in X$ so for each y in X s.t $x \notin y$ there exists $U, V \in R.g.o(X)$ s.t $x \in U \subseteq \tilde{\tilde{y}}$ and $\tilde{y} \in V \subseteq \tilde{\tilde{x}}$ .Since $\tilde{x} \in U$ so $x \in U$ we have to prove that $\tilde{x}$ is R.g.o .

That is to prove that $\tilde{x}$ is R.g.o(X) for if $\tilde{\tilde{x}} = U\{V : \tilde{y} \in V, V \in R.g.o(X)\}$ .that is $\tilde{x}$ is union of R.g.o set so it is R.g.o therefor $\tilde{x}$ is R.g.C. RGT1(iv) $\Rightarrow$RGT1(viii)

Suppose that RGT1(iv) hold and let $x \in X$ so for each $y \in X$ s.t $x \neq y$ there exists U,V ∈R.g.o(X) s.t $\tilde{\tilde{y}} \notin U$ and $\tilde{\tilde{x}} \notin V$ .We have to prove that $\tilde{x}$ is R.g.c .That is we have to prove that $\tilde{\tilde{x}}$ is R.g.o(X).

Since $\tilde{\tilde{x}} = \langle x, \{x\}^c, \phi \rangle = U\{U : \tilde{\tilde{y}} \notin U\} so, \tilde{\tilde{x}}$ is aunion of R.g.o'S So is R.g.o therefore $\tilde{\tilde{x}}$ is R.g.C.

The following implication are proved by transitivity.

RGT1(ii) + RGT1(i)$\Rightarrow$RGT1(vi),
RGT1(ii) + RGT1(i)$\Rightarrow$RGT1(v) ,
RGT1(ii) + RGT1(i)$\Rightarrow$RGT1(iv) and
RGT1(ii) + RGT1(i)$\Rightarrow$RGT1(viii)

The converse of theorem 3.2 are not true in general. The following examples show these cases.

Example **: 3.3**

1-Let X ={a,b} and define T={$\tilde{\phi}$, $\tilde{X}$,A,B } , where A=<x, ϕ,{a}> , B = < x , ϕ, ϕ> , so R.C(X) (where $B = B^{\bar{o}}$) = { $\tilde{\phi}, \tilde{X}, B$} then R.g.o(X) = T , so the IT(X,T) satisfies RGT1(V) , but dose not satisfy RGT1(i).

2-let X={a,b} and define T={$\tilde{\phi}, \tilde{X}, A, B, C$}, where A<$x$,{a},$\phi$> ,B=<$x$,{b}$\phi$> ,C = <$x$, ϕ,ϕ> , so R.C(X)={ $\tilde{\phi}$, $\tilde{X}$,c}and R.g.O(X)=T , so the IT(X,T) satisfies RGT1(vi) , but not satisfies RGT1(ii).

3-Take X={a,b} and define T={$\tilde{\phi}$, $\tilde{X}$,A,B,C} , where A=<x, ϕ,{a}> , B=<x, ϕ,ϕ>, C=<x,{a} , ϕ> , so R.C(X)=T and R.g.o(X)=T$\cup${E,G} , where E=<x,{b},ϕ> , G=<x,{b},{a}> , so the IT(X,T) satisfies RGT1(viii) but not satisfies RGT1(iv) and satisfies RGT1 (vii) but not satisfies RGT1(iii).

4-Take X={a,b} and defined T={$\tilde{\phi}$, $\tilde{x}$,A,B,C} , where A=<x, ϕ,ϕ> , B=<x, ϕ,{b}> , C=<x, ϕ,ϕ> so R.c(X) = {$\tilde{\phi}$, $\tilde{X}$,C} and R.g.O (X) = T , so the IT(X.T) satisfies RGT1(iv) , but not satisfies RGT1(ii).

5-Let X={a,b} and define T={$\tilde{\phi}$, $\tilde{X}$,A,B,C} where A=<x,{a},{b}> , B=<x,{b}ϕ>, C=<x, ϕ,{b}> , so R.C(X)={ $\tilde{\phi}$, $\tilde{X}$,B,C} and R.g.O(X)={

$\tilde{\phi}$, $\tilde{X}$,A,B,C,E} where E=< X ,{a}, ϕ > then the IT (X,T) satisfies RGT1(i) but not satisfies RGT1(ii).

4.The Separation axiom Regular generalized T_2:

In this section we recall the definition of weak forms of the separation axiom namely regular generalization T2(ki) (RGT2(k) for short) , where k∈(i ,ii , iii , iv, v ,vi) in ITS.

Definition : 4.1

Let (X,T) be an ITS,(X,T) is said to be :

a)RGT2(i) if for each $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{x} \in U$, $\tilde{y} \in V$ and $U \cap V = \tilde{\phi}$.

b) RGT2(ii) if for each $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{\tilde{x}} \in U$, $\tilde{\tilde{y}} \in V$ and $U \cap V = \tilde{\phi}$.

c) RGT2(iii) if for each $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{x} \in U$, $\tilde{y} \notin V$ and $U \subseteq \bar{V}$.

d) RGT2(iv) if for each $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{\tilde{x}} \in U$, $\tilde{\tilde{y}} \in V$ and $U \subseteq \bar{V}$.

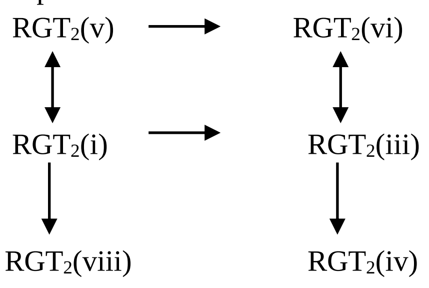
e) RGT2(v) if for each $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{x} \in U \subseteq \bar{\tilde{y}}, \tilde{y} \in V \subseteq \bar{\tilde{x}}$ and $U \cap V = \tilde{\phi}$.

f) RGT2(vi) if for each $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{\tilde{x}} \in U \subseteq \bar{\tilde{y}}, \tilde{\tilde{y}} \in V \subseteq \bar{\tilde{x}}$ and $U \cap V = \tilde{\phi}$.

The following in the main theorem it gives relations of the several kinds of RGT2 separation axioms.

Theorem : 4.2

Let (X,T) be an ITS.Then the following implications are valied :



: RGT_2(v) → RGT_2(vi)
Let (X,T) be an ITS satisfy

Proof

RGT2(V) , for if let $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{x} \in U$. $\tilde{y} \notin U$ and $U \cap V = \tilde{\phi}$ .Since $\tilde{x} \in U$ and $\tilde{y} \in V$, then we can get easily that $\tilde{\tilde{x}} \in U$ and $\tilde{\tilde{y}} \in V$, therefore $\tilde{\tilde{x}} \in V$ and $\tilde{\tilde{y}} \in U$ and $U \subseteq \bar{\tilde{y}}, V \subseteq \bar{\tilde{x}}$ and $U \cap V = \tilde{\phi}$ .So we get that (X,T) is satisfy RGT2(Vi) , RGT2(i)⇒RGT2(ii).

Let (X,T) be an ITS satisfy RGT2(i) so take $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{x} \in U$.$\tilde{y} \in V$ and $U \cap V = \tilde{\phi}$ .Since $\tilde{x} \in U$ and, $\tilde{y} \in V$ then we can get easily that $\tilde{\tilde{x}} \in U$ and $\tilde{\tilde{y}} \in V$, and $U \cap V = \phi$ .from hypothesis .Therefore RGT2(ii) holds.

RGT2(i)⇒RGT2(iii).

Let (X,T) be ITS satisfy RGT2(i) , for if $x,y \in X, x \neq y$ .Since RGT2(i) holds , this implies that there exists $U,V \in R.g.o(X)$ s.t $\tilde{x} \in U$. $\tilde{y} \in V$ and $U \cap V = \tilde{\phi}$ .Since $\tilde{x} \in U$ and $U \cap V = \tilde{\phi}$, so $\tilde{x} \notin V$, this implies that $\tilde{x} \in \bar{V}$ .This prove that for every x in X if $\tilde{x} \in U$, then $\tilde{x} \notin V$ i.e $U \in \bar{V}$
Therefore (X,T) satisfies RGT2(iii) .

RGT2(iii)⇒RGT2(i)

Let (X,T) be an ITS satisfies RGT2(iii) so there exists $U,V \in R.g.o(X)$ such that $\tilde{x} \in U$, $\tilde{y} \in V$ and $U \subseteq \bar{V}$ .To prove that $U \cap V = \tilde{\phi}$ .Since $U \subseteq \bar{V}$ and $\tilde{x} \in U$ so $x \in \bar{V}$, this implies that $x \notin V$ .Therefore $U \cap V = \phi$ so (X,T) satisfies RGT2(i).

RGT2(ii)⇒RGT2(iv)

Since RGT2(ii) hold , so let $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{\tilde{x}} \in U$, $\tilde{\tilde{y}} \in V$ and $U \subseteq \bar{V}$. So $\tilde{\tilde{x}} \in \bar{V}, \tilde{\tilde{y}} \in V$ and $U \cap V = \phi$. so $\tilde{\tilde{x}} \in \bar{V}$, then $\tilde{\tilde{x}} \in V$ and $U \cap V = \tilde{\phi}$, so $\tilde{\tilde{x}} \in U$, therefore $U \subseteq \bar{V}$ that is mean RGT2(iv) holds.

RGT2(vi) ⇒RGT2(ii)

Let $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ since RGT2(vi) holds so

$\tilde{\tilde{x}} \in U \subseteq \bar{\tilde{y}}$, $\tilde{\tilde{y}} \in V \subseteq \bar{\tilde{x}}$ and $U \cap V = \tilde{\phi}$ from this we directly that there exists $U,V \in R.g.o(X)$, s.t $\tilde{\tilde{x}} \in U$, $\tilde{\tilde{y}} \notin V$ and $U \cap V = \tilde{\phi}$, therefore RGT2(ii) holds.

RGT2(iV)$\Rightarrow$RGT2(i) is clear

RGT2(iii)$\Rightarrow$RGT2(iv)

Let (X,T) be an ITS satisfies RGT2(iii) , to Prove that (X,T) satisfies RGT2(iv) , for if $x,y \in X, x \neq y$ .Since RGT2(iii) holds , this implies that there exists $U,V \in R.g.o(X)$ such that $\tilde{x} \in U$. $\tilde{y} \notin V$ and $U \in \bar{V}$ .so we get directly that $\tilde{x} \in U$, $\tilde{\tilde{y}} \in V$ and $U \in \bar{V}$. Therefore (X,T) satisfies RGT2(iv).

RGT2(v) $\Rightarrow$RGT2(i)

Let (X,T) be an ITS satisfies RGT2(v) for if $x,y \in X, x \neq y$, so there exists $U,V \in R.g.o(X)$ such that $\tilde{\tilde{x}} \in U \subseteq \bar{\tilde{y}}$, $\tilde{y} \in V \subseteq \bar{\tilde{x}}$ and $U \cap V = \tilde{\phi}$ from this we get directly that $\tilde{\tilde{x}} \in U$, $\tilde{\tilde{y}} \in V$ and $U \cap V = \tilde{\phi}$ .Therefore (X,T) satisfies RGT2(i).

RGT2(i) $\Rightarrow$RGT2(v)

Let (X,T) be an ITS satisfies RGT2(i),to prove that (X,T) satisfies RGT2(v), for if $x,y \in X, x \neq y$ .Since RGT2(i) holds , this implies that there exists $U,V \in R.g.o(X)$ such that $\tilde{\tilde{x}} \in U$, $\tilde{\tilde{y}} \in V$ and $U \cap V = \tilde{\phi}$ .we have to prove $U \subseteq \bar{\tilde{y}}$ and $V \subseteq \bar{\tilde{x}}$ i.e .U1$\subseteq \{y\}^c$ and {y}$\subseteq$U2 also V1 $\subseteq \bar{\tilde{y}}$ .Let U=<x,U1,U2> and $\bar{\tilde{y}} \subseteq U_2$ also $V_1 \subseteq \{x\}^c$ and $\{x\} \subseteq V_2$ .Frstly it is prove $U \subseteq \bar{\tilde{y}}$. Let U=<$x$,U1,U2> and $\bar{\tilde{y}} =< x, \{y\}^c, \{y\} >$, let $\tilde{z} \in U$ this implies that $z \in U_1$ .Since $\tilde{y} \in V$ and $U \cap V = \tilde{\phi}$ this implies that $\tilde{y} \notin U$ so $\tilde{y} \notin V_1$, if $\tilde{z} \notin \tilde{y}$ this implies that $\tilde{z} \in \tilde{y}$ and so $z \in \{y\}$ .Therefore $U_1 \subseteq \{y\}$ .if $\tilde{z} \in \tilde{y}$ this implies that $z \in \{y\}$ we get that a contradiction (because $z \in \{y\}$), hence $\tilde{z} \notin \tilde{y}$ .Therefore $U_1 \subseteq \{y\}$. Now we have to prove $\{y\} \subseteq U_2$. Let $y \in \{y\}$ this implies that $y \notin \{y\}$, so $y \notin U_1$ hence $y \in V_2$.So $\{y\} \subseteq U_2$.

In a simlier way , we can prove $V \in \tilde{x}$ .Therefore (x,T) satisfies RGT2(V).

RGT2(ii) $\Rightarrow$RGT2(iv)

Let (X,T) be an ITS satisfies RGT2(ii) this implies that there exists U,V$\in$ R.g.o(x).

Such that $\tilde{\tilde{x}} \in U \subseteq \bar{\tilde{y}}$, $\tilde{\tilde{y}} \in V \subseteq \bar{\tilde{x}}$ and $U \cap V = \tilde{\phi}$ we have to prove that $U \in \bar{V}$ and $V \subseteq \bar{\tilde{x}}$ i.e. V1 $\subseteq \bar{\tilde{y}}$ and $\phi \subseteq U_2$ also $V_1 \subseteq \{x\}^c$ and $\phi \subseteq U$ .Firstly it is prove $U \subseteq \bar{\tilde{y}}$ .Let U=<x,U1,U2> and $\bar{\tilde{y}} =< x, \{y\}^c, \phi >$ .Since $\phi \subseteq U_2$ we have to prove $U \subseteq \{y\}^c$.

Let $\tilde{z} \in U$ this implies that $z \notin U_2$, so $z \in U_1$ ($U_1 \cap U_2 = \phi$).Since $\tilde{y} \notin U$ ($\tilde{y} \in V$ and U$\cap$ V) , this implies that $\tilde{\tilde{z}} \notin \tilde{\tilde{y}}$ .Hence $\tilde{\tilde{z}} \in \bar{\tilde{y}}$ so $z \in \{y\}^c$ .Therefore $U_1 \subseteq \{y\}^c$.

In a similar way , we can prove $V \subseteq \bar{\tilde{x}}$ .So (x,T) satisfies RGT2(iv).

The following implications followed from theorem 2.2 by transitivity.

RGT2(v)$\Rightarrow$RGT2(ii) , RGT2(i)$\Rightarrow$RGT2(iv)

RGT2(v)$\Rightarrow$RGT2(vi) , RGT2(vi)$\Rightarrow$RGT2(iv)

RGT2(v)$\Rightarrow$RGT2(iii)

In general the converse of the diagram appears in the theorem 4.2 is not true in general .The following counter example shows the cases.

Example: 4.3

(1)Let X={a,b} and T={$\tilde{\phi}, \tilde{X}, A, B, C$} , where A=<$x$,$\phi$,{a}> , B=< x,ϕ, ϕ> so Rc(X)={ $\tilde{\phi}, \tilde{X}, C$ } and R.g.o.(X)=T , so the IT(X,T) satisfies RGT2(iv) , but not satisfies RGT2(ii).

(2)Let X={a,b,c} and define T={$\tilde{\phi}, \tilde{X}, A, B, C$,D,E,F,G,H} where A=<$x$,{a,b},{c}> , B=<$x$,{a},{b,c}>, C=< x,{b},{a,c}>, D=< x,{c},{a}> , E=< x,{a,b}, ϕ > , F=< x,{b,c, ϕ > , H=< x, ϕ , ϕ >.

RC(X) = { $\tilde{\phi}$, $\tilde{X}$,E,H} and R.g.o(X)=TU{J,N,O,Q,V} where J=< x,{b,c},{a} , N=< x, ϕ ,c> , O=< x,{b},{a}> , Q=< x,{b}, ϕ > , V=< x, ϕ , ϕ > , so that IT(X) satififies RGT2(i) , but not satisfies RGT2(vi) and not satisfies RGT2(v)

***Corollary* : 4.4**

Let(X,T) be ITS , then if (X,T) satisfies RGT2(k) , then it satisfies RGT1(k) , where $k \in (i, ii, iii, iv, v, vi)$.

Remark: 4.5

The converse of corollary 4.4 is not true in general .The following examples in example 3.3 showes these cases.

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بعض بديهيات الفصل المعممة المنتظمة في الفضاءات التبولوجية الحدسية

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الخلاصة

الهدف من هذا البحث هو اعطاء تعريف جديد لبعض بديهيات الفصل المعممة المنتظمة في الفضاءات التبولوجية الحدسية ودراسة بعض العلاقات التي تربط بديهيات الفصل المعممة المنتظمة اضافة الى دراسة بعض الخواص والعلاقات وتعميمها مع الامثلة التوضيحية.