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Modification of Three Order Methods For Solving Satellite Orbital Equation in Elliptical Motion

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ABSTRACT

In the present study, a modification for iterative methods of three order is presented by means of mean anomaly to calculate the values of E and e (true anomaly and eccentricity) respectively, for a planet in an elliptical orbit around the sun. We find that the improved methods converge to the true anomaly value E, solution with less iteration. The efficiency of the modified third order algorithms are examined on many cases of the values of M and e. It is observed that methods are more efficient than third order methods presented by Weerakoon [1], Homerier [2] and Ababneh [3] and classical Newton's method.

1. INTRODUCTION

In the engineering and scientific applications, one of the popular important problems is to calculate the solution for non-linear equations. The boundary value problems appear in elasticity and the kinetic phenomena of gases as well as the voltage difference expressions in the transonic system of dense gases in gas dynamics may be reduced to nonlinear equations [4-5]. Astronomy has a source of researcher's elevation. The greatest revolution in engineering and science is stimulated by astronomy. Since ancient times, civilizations have questioned the movement of the laws for the celestial bodies. One of the problems that appear in celestial mechanical literature is the solution of equation. It gives the relationship between the polar coordinates of a celestial body and the elapsed time from a given initial point. In celestial mechanics, Kepler's equation is an importance. It cannot be directly reversed in terms of simple functions in order to locate the planet at a given time [6-8]. Earth's motion means that; the earth revolves in an elliptical orbit around the sun, which is a focus of the elliptical ellipse, the value of it's eccentricity is about 0.0167. The full time for Earth orbit to reach the sun is 365 days. In general, the earth approaches the sun is the 4th day of January of each year. In order to acquire the earth's position at this time requires; the solving of Kepler's equation of elliptical motion should be achieved. In this case, $e = 0.0167$, $T = 365$ days, $t = 27$ days, therefore; the mean anomaly $M = \frac{2\pi t}{T}$, (where T: is the period of the motion and M : is the time since periapsis. $M = 2 \times (3.14159) \times \frac{27}{365} = 0.46478$ in radians. Obtaining the earth's at the 1st of February needs to finding the equation of an elliptical motion using the eccentric anomaly E,

$$E - esin(E) = M \quad \dots (1)$$

where $e \in [0, 1)$ and $M \in [0, \pi)$.

A unique solution has been obtained by means of equation, when M is not multiple of π since equation cannot be solved directly for E and M; therefore some algorithms derived for obtaining the solution of such equation [9-16]. The iterative techniques are one of the important techniques to treat transcendental equation. Among these methods are the functional iteration, the Newton–Raphson, and other methods [17-25]. In order to reduce the number of iterations in the solution of a nonlinear equation, one must improve the iterative methods by increasing the order of convergence. In [26] Aisha et al. used Kepler equation in terms of the eccentric anomaly of a planet that orbits the Sun, convergence of the obtained approximate solutions and highly accurate numerical results have been obtained. In

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[27] Aisha et al. used the whole domain of eccentricity and mean anomaly in order to demonstrate the obtained numerical solutions for the planets in the solar system based on Kepler equation. In [28] Calvo et al. iterative solution of the sine hyperbolic Kepler's equation (SHK) has been investigated using numerical examples. In [29] Badolati et al. they examined some approximate solutions for Kepler's equation, these explicit formulae, obtained by Trembley, Pacassi, Fergola, and Horrebow. They proved that the formulae's are equivalent and give an evaluation of the error. In [30] Daniele et al. a novel theoretical analysis of the errors that agrees with the numerical results based on a universal algorithm have been obtained. Moreover; for Kepler problem, it's python code is ~ 2000 times faster than Newton methods for large number of point N.

The purpose of the present work is adopted an modified accelerated method which is given in the next section and compared the obtained results with other third order convergence methods to find the solution of satellite orbital equation in elliptic motion, by considering several cases of different M and e values such that $e \in [0,1)$ and $M \in [0,\pi)$.

2. SOLUTION of ELLIPTIC ORBIT EQUATION

Many researchers developed the order of convergence of Newton's in a number of different ways to modify the order of convergence for Newton's method. In the present article, the researcher develops the following third-order methods for solving Kepler's equation in elliptical orbit, Eq. 1. The third order methods derived by Weerakoon [1], Homerier [2] and Ababneh [3] are respectively,

$$u_{n+1} = u_n - \frac{2f(u_n)}{f'(u_n)+f'(u_n-f(u_n)/f'(u_n))} \quad \dots (2)$$

$$u_{n+1} = u_n - \frac{f(u_n)}{f'(u_n-f(u_n)/2\,f'(u_n))} \quad \dots (3)$$

$$u_{n+1} = u_n - \frac{f(u_n)(f'(u_n)+f'(u_n-\frac{f(u_n)}{f'(u_n)}))}{f'(u_n)} \quad \dots (4)$$

Note that, three functions and first derivative evaluation are needed in Eqns. 2-2 to 2-4.

The suggested modification algorithms are the development of Eqns. 2-2 to 2-4 to solving Eqn. 1 are given by the following three algorithms.

2.1. Modified Weerakoon Method: MWM Algorithm

The algorithm of the Modified Weerakoon method is illustrated by the following equation

$$u_{n+1} = u_n - \frac{2f(u_n)}{f'(u_n) + f'\left(u_n - f(u_n)/f'(u_n)\right)}$$

INPUT: Value of mean anomaly M.
Maximum number of iterations I_m
Value of the eccentricity e,
Initial true anomaly $E_o = M + 0.85\,e$ ...(5)
$t = 0$
OUTPUT: Approximate solution of the true anomaly E^*_{t+1}.
Step 1: Do steps $3-4$ while; $t \le I_m$; where I_m is maximum number of iteration
Step 2: $f = M - E_o + e\sin E_o$
$\underline{f_1 = e\cos E_o - 1}$ ... (6)
$y = E_o - \frac{f}{f_1}$ ... (7)
$f_2 = e\cos y - 1$ ... (8)
$E_{t+1} = E_t - \frac{2\,f}{f_1+f_2}$ ...(9)
$E^*_{t+1} = E_t - \frac{(E_{t+1}-E_t)^2}{E_{t+2}-2\,E_{t+1}+E_t}$ ...(10)
Step 3: If $|E^*_{t+1} - E^*_t| < 10^{-15}$, then output $\overline{E_{t+1}}$ and stop
Step 4: Put $t = t+1$ and goto step 1.

2.2. Modified Homerier Method: MHM Algorithm

The algorithm of the Modified Homerier method is illustrated by the following equation

$$u_{n+1} = u_n - \frac{f(u_n)}{f'\left(u_n - f(u_n)/2\,f'(u_n)\right)}$$

INPUT: Value of mean anomaly M.
Maximum number of iterations I_m
Value of the eccentricity e,
Initial true anomaly $E_o = M + 0.85$ ...(11)
$t = 0$
OUTPUT: Approximate solution of the true anomaly E^*_{t+1}.

Step 1: Do steps 2, 3 and 4 while $t \le I_m$
Step 2: $f = M - E_o + e\sin E_o$ ...(12)
$f_1 = e\cos E_o - 1$ ... (13)
$y = E_o - \frac{f}{2\,f_1}$ ...(14)
$f_2 = e\cos y - 1$ ... (15)
$E_{t+1} = E_t - \frac{f}{f_2}$ ...(16)
$E^*_{t+1} = E_t - \frac{(E_{t+1}-E_t)^2}{E_{t+2}-2\,E_{t+1}+E_t}$ ...(17)
Step 3: If $|E^*_{t+1} - t| < 10^{-15}$, then output $\overline{E_{t+1}}$ and stop
Step 4: Put $t = t+1$ and goto step 1.

2.3. Modified Ababneh Method: MAM Algorithm

The algorithm of the Modified Ababneh method is illustrated by the following equation

$$u_{n+1} = u_n - \frac{f(u_n)(f'(u_n) + f'(u_n - \frac{f(u_n)}{f'(u_n)}))}{f'(u_n)}$$

INPUT: Value of mean anomaly M.
Maximum number of iterations I_m
Value of the eccentricity e,
Initial true anomaly $E_o = M + 0.85\,e$ ...(18)
$t = 0$
OUTPUT: Approximate solution of the true anomaly E^*_{t+1}.
Step 1: Do steps 3 and 4 while $t \le I_m$
Step 2: $f = M - E_o + e\sin E_o$ ... (19)
$f_1 = e\cos E_o - 1$ ...(20)

$$y = E_o - \frac{f}{f_1} \quad \ldots(21)$$

$$f_2 = e\cos y - 1 \ldots (22)$$

$$E_{t+1} = E_t - \frac{f(f_1 + f_2)}{(f_1)^2 + (f_2)^2} \quad \ldots(23)$$

$$E^*_{t+1} = E_t - \frac{(E_{t+1} - E_t)^2}{E_{t+2} - 2\,E_{t+1} + E_t} \quad \ldots(24)$$

Step 3: If $|E^*_{t+1} - E^*_t| < 10^{-15}$, then output $\overline{E_{t+1}}$; stop

Step 4: Set $t = t + 1$ and goto step 1.

3. RESULTS AND DISCUSSION

We present some test results that are presented to solve eq. 1 using a different couple of values for the mean anomaly (M) and eccentricity (e) to find approximate values for the true anomaly (E). Numbers of iterations are given in Table 1, while Table 2 illustrates the approximate solution of the true anomaly E for the couple values (M, e). The results are compared against the Newton's method (NM), Weerakoon method (WM), Eq. 2, the Homeier's method (HM), Eq. 3 Kou method (KM), Eq. 4, and our modified methods, described in Modified Weerakoon Method (MWM), Modified Homerier Method (MHM) and Modified Ababneh Method (MAM) algorithms. The computations are done with the aid of Matlab. An approximate solution of the true anomaly E is accepted based on the precisionϵ. The stopping criteria $|E_{n+1} - E_n| < 10^{-15}$ is utilized for computer programs.

Table 1: Number of iterations required using various methods

Number of Iterations								
Values of Mean Anomaly and Eccentricity		NM	WM	HM	AM	MWM	MHM	MAM
M	e							
0.01	0.01	8	4	3	3	2	1	1
0.08	0.08	9	4	3	3	2	1	1
0.05	0.5	6	6	4	4	4	2	2
0.001	0.95	9	6	5	5	4	3	3
0.05	0.95	7	5	4	4	3	2	2
0.099	0.95	8	6	3	3	4	1	1
0.001	0.95	9	7	4	4	5	2	2
0.05	0.5	6	5	4	4	3	2	2
0.08	0.95	7	5	4	4	3	2	2
0.07	0.95	7	5	4	4	3	2	2
0.07	0.09	6	4	3	3	2	1	1
0.06	0.95	7	5	4	4	3	2	2
0.06	0.09	6	4	3	3	2	1	1
0.0005	0.09	5	4	3	3	2	1	1
2	0.1	6	4	3	3	2	1	1
2	0.2	5	4	3	3	2	1	1
2	0.3	5	4	3	3	2	1	1
2	0.4	6	4	3	3	2	1	1
2	0.5	6	4	3	3	2	1	1
2	0.6	6	4	3	3	2	1	1
2	0.7	6	4	3	3	2	1	1
2	0.8	6	4	3	3	2	1	1
2	0.9	6	4	3	3	2	1	1
0.01	0.99	9	6	5	5	4	3	3
0.01	0.98	9	6	5	5	4	3	3
0.01	0.97	9	6	5	5	4	3	3
1.6	0.5	6	4	3	3	2	1	1
1.8	0.5	6	4	3	3	2	1	1
0.2	0.8	6	4	3	3	2	1	1
0.4	0.8	6	4	3	3	2	1	1
0.6	0.8	6	4	3	3	2	1	1
0.8	0.8	6	4	3	3	2	1	1
1	0.8	6	4	3	3	2	1	1
1.2	0.8	6	4	3	3	2	1	1
1.4	0.8	6	4	3	3	2	1	1
1.6	0.8	6	4	3	3	2	1	1
1.8	0.8	6	4	3	3	2	1	1
1.6	0.5	6	4	3	3	2	1	1
1.8	0.5	6	4	3	3	2	1	1
0.2	0.4	5	4	3	3	2	1	1
0.8	0.4	5	4	3	3	2	1	1
0.2	0.3	5	4	3	3	2	1	1
0.6	0.3	5	4	3	3	2	1	1
0.2	0.2	5	4	3	3	2	1	1
0.3	0.2	5	4	3	3	2	1	1

Table 2: The computed approximate values of the true anomaly E^*

Values of Mean Anomaly and Eccentricity		Approximate Value of True Anomaly E^*
M	e	
0.01	0.01	0.010101008365986
0.08	0.08	0.086946999248733
0.05	0.5	0.099834243368842
0.001	0.95	0.019974762949968
0.05	0.95	0.531420642466566
0.099	0.95	0.738568830061134
0.001	0.95	0.019974762949968
0.05	0.5	0.099834243368842
0.08	0.95	0.669752004546650
0.07	0.95	0.628653087236876
0.07	0.09	0.076915578596776
0.06	0.95	0.583020838445193
0.06	0.09	0.065929343219182
0.0005	0.09	5.49450546716e-004
2	0.1	2.086971338731819
2	0.2	2.165646494384257
2	0.3	2.236031495172437
2	0.4	2.298643367069317
2	0.5	2.354242758222781
2	0.6	2.403657147257398
2	0.7	2.447683214615955
2	0.8	2.487042160818273
2	0.9	2.522365434000245
0.01	0.99	0.342270316491775
0.01	0.98	0.293794336770122
0.01	0.97	0.249697938962838
1.6	0.5	2.044860223886352
1.8	0.5	2.203280684852710
0.2	0.8	0.738606649521026
0.4	0.8	1.120123336811002
0.6	0.8	1.386444170341408
0.8	0.8	1.599666625497567
1	0.8	1.782191328937901
1.2	0.8	1.944720812115405
1.4	0.8	2.093269946673696
1.6	0.8	2.231599587305797
1.8	0.8	2.362249720006130
1.6	0.5	2.044860223886352
1.8	0.5	2.203280684852710
0.2	0.4	0.495007682285571
0.8	0.4	1.167985034818690
0.2	0.3	0.249355982235956
0.6	0.3	0.733963591129846
0.2	0.2	0.222020054224999
0.3	0.2	0.442851789043066

4. CONCLUSION

Through comparing the proposed modified algorithms with other methods, the results of Table 1 show that our modified algorithm converge faster to the approximate values of E (true anomaly) in small numbers of iterations for each values of M and e such that $0 \leq e \leq 1$ and $0 \leq M \leq 1$, starting with the initial solution $E_o = M + 0.85\,e$. Large iterations are needed to converge to the solution with the cases when M near $zero$ and e near 1 as illustrated in Table 1 for the couple of (M, e) we have $(0.001,0.95)$, $(0.01, 0.99)$, $(0.01, 0.98)$ and $(0.01, 0.97)$ require more large number of iterations to get the solution. In Table 2 the approximate values of true anomaly values E^* [0.0005-2.5] based on different values of mean anomaly M [0.0005-2] and eccentricity e [0.01-0.2] have been obtained. The maximum value of E^* 2.522 is obtained when M =0.0005 and e=0.01; while the minimum value of E^* 0.0006 is obtained when $M = 2$ and e=0.99;

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تطوير طرق من الرتبة الثالثة لحل معادلة مدار القمر الصناعي خلال حركته في مدار القطع الناقص

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الخلاصة:
في هذه البحث، تم تقديم تطوير لبعض الطرق التكرارية من الرتبة الثالثة لأيجاد قيمة الانحراف الحقيقي E من قيمة متوسط الانحراف M وقيمة الانحراف الشاذ e ، لأي كوكب موجود في مسار القطع الناقص يدور حول الشمس (معادلة كبلر). لغرض تقليل عدد التكرارات وجدنا أن الطرق المُطورة تقترب من قيمة الانحراف الحقيقي بأقل تكرار. تم فحص كفاءة الطرق المُطورة من الرتبة الثالثة بعدة حالات من قيم M و e. لوحظ أن الطرق المقترحة أكثر كفاءة من طريقة نيوتن التقليدية والطرق المُقدمة من قبل Weerakoon [1] , Homerier [2] ، و Ababneh [3] .