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Numerical Methods on the Triple Informative Prior Distribution for the Failure Rate Basic Gompertz Model

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ABSTRACT

In this paper, it has been dealt with basic Gompertz distribution. The maximum likelihood, Bayes methods of estimation were used to estimate the unknown shape parameter. The failure rate (hazard) function with the least loss was found using different priors (Gamma, exponential, chi-square and triple prior) under symmetric loss function (Degroot loss function). A comparison was made about the performance of these estimators with the numerical solution that was found using expansion methods (Bernstein polynomial and power function) which was applied to find the failure rate function numerically. The proficiency test of the proposed methods was conducted with a number of test examples. Finally, for computations the Matlab (R2015b) is used.

1. INTRODUCTION

Benjamin Gompertz (1825) has proposed the basic Gompertz (BG) distribution [1]. It is important in describing the pattern of adult deaths and actuarial tables. The Gompertz distribution has many real life applications, especially medical and actuarial studies. In addition, it used as a survival model in reliability and failure rate (Hazard) [2].

The probability density function (pdf) and cumulative distribution function of (BG) random variable given as [3]:

$$f(t;\lambda) = \lambda \exp[t + \lambda(1 - e^t)] \;;\; t \geq 0 \,, \quad \ldots (1)$$

$$F(t;\lambda) = 1 - \exp[\lambda(1 - e^t)] \;;\; t \geq 0 \,, \quad \ldots (2)$$

where $\lambda > 0$ is the shape parameter.

The corresponding reliability function, R(t), and failure rate function, h(t), at mission time t are given as [3, 13]:

$$R(t) = \exp[\, \lambda(1 - e^t) \,] \;;\; t \geq 0 \,, \quad \ldots (3)$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda \exp[t + \lambda(1 - e^t)]}{\exp[\, \lambda(1 - e^t)]} = \lambda\, e^t \quad \ldots (4)$$

The basic Gompertz distribution belongs to the exponential family of lifetime distribution, the Gompertz function is a type of mathematical model for a time series, and this function originally designed to describe human mortality but since modified to be applying in biology, with regard to detailing populations. It has been introduced by many authors some of whom are Gompertz [1] is formulating the Gompertz model for human composition mortality and greeting actuarial tables. Lenart [4] derived the maximum likelihood estimators of the parameters that included the moments of the Gompertz distribution. Sanku Dey etal. [5] Discuss the statistical properties and different methods of estimation of the Gompertz distribution with application. Hanaa and Nouf [6] study the estimation of some unknown parameters of the Beta exponential Gompertz distribution using complete samples. Garg etal. [7] Study the properties and the maximum likelihood estimators of the parameters of the Gompertz distribution on mortality in mice.

In this work, the informative priors three single priors and one triple prior with De-groot loos function are used to find the failure rate function. As well as, numerical method (expansion methods: Bernstein polynomial and power

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function) [8, 9, 10], are used to estimation the failure rate function h(t), in this method expanding h(t) in terms of a set of power function as in [11,12] to find approximate estimation of h(t), and then comparison between the exact and all estimator using mean square errors(MSE).

2. MAXIMUM LIKELIHOOD ESTIMATOR

Let $\underline{t} = (t_1, t_2, \ldots, t_n)$ be the life time of a random sample of size n drawn independently from basic Gompertz distribution defined by (1). The likelihood functions for the given sample an observation defined as [13]:

$$L(\lambda \mid \underline{t}) = \prod_{i=1}^{n} f(t_i \mid \lambda) = \lambda^n e^{[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1-e^{t_i})]} \quad \ldots (5)$$

$$\ln(L) = n \ln\lambda + \sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1 - e^{t_i})$$

The maximum likelihood estimator (MLE) of unknown shape parameter λ, denoted by $\hat{\lambda}_{ML}$ yields by taking the derivative of natural log likelihood function with respect to λ and setting it equal to zero as:

$$\hat{\lambda}_{ML} = \frac{-n}{k} \quad \text{where} \quad k = \sum_{i=1}^{n}(1 - e^{t_i}) \quad \ldots(6)$$

The MLE's of R(t) and h(t), based on the invariant property of the MLE are defined as:

$$\hat{R}_{ML} = \exp[\hat{\lambda}_{ML} (1 - e^{t})] \quad \ldots (7)$$

$$\hat{h}_{ML}(t) = \hat{\lambda}_{ML}\, e^{t} = \frac{-n\, e^{t}}{\sum_{i=1}^{n}(1 - e^{t_i})} \quad \ldots (8)$$

2.1. Bayes Estimator

From Baye's rule the posterior pdf of unknown parameter λ, results by combining likelihood function $L(\lambda \mid \underline{t})$ with density function of prior distribution $g(\lambda)$ as[14]:

$$\pi(\lambda \mid \underline{t}) = \frac{L(\lambda \mid \underline{t})\, g(\lambda)}{\int_{\lambda} L(\lambda \mid \underline{t})\, g(\lambda)\, d\lambda} \quad \ldots (9)$$

The most widely used prior distribution of the parameter λ is the gamma distribution with hyper-parameter 'a' and 'b' with pdf given by [15].

$$g_1(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda} \quad ; \lambda > 0 \text{ and } a, b > 0 \quad \ldots (10)$$

The posterior distribution of the unknown parameter λ of BG have been obtained by substitute eq.(5) and eq.(10) in eq.(9):

$$\pi_1(\lambda \mid \underline{t}) = \frac{\lambda^n e^{[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1-e^{t_i})]} \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}}{\int_0^{\infty} \lambda^n e^{[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1-e^{t_i})]} \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}\, d\lambda}$$

$$= \frac{\lambda^{n+a-1}\, e^{-\lambda(b-k)}}{\int_0^{\infty} \lambda^{n+a-1}\, e^{-\lambda(b-k)}\, d\lambda}$$

where $k = \sum_{i=1}^{n}(1 - e^{t_i})$

By using the transformation $y = \lambda(b-k)$ and $\lambda = y/(b-k)$ and the derivative with respect to y is $d\lambda = dy/(b-k)$ then we obtain the final formula as:

$$\pi_1(\lambda \mid \underline{t}) = \frac{(b-k)^{n+a}\, \lambda^{n+a-1}\, e^{-\lambda(b-k)}}{\Gamma(n+a)} \quad \ldots (11)$$

The second prior distribution exponential distribution with hyper-parameter 'a' having pdf given by [16].

$$g_2(\lambda) = a\, e^{-a\lambda} \quad ; \lambda > 0 \text{ and } a > 0 \ldots (12)$$

The posterior distribution of the unknown parameter λ of BG have been obtain by combining eq. (5) and eq. (9) with eq.(12) as:

$$\pi_2(\lambda \mid \underline{t}) = \frac{\lambda^n e^{[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1-e^{t_i})]}\, a\, e^{-a\lambda}}{\int_0^{\infty} \lambda^n e^{[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1-e^{t_i})]}\, a\, e^{-a\lambda}\, d\lambda} = \frac{\lambda^n\, e^{-\lambda(a-k)}}{\int_0^{\infty} \lambda^n\, e^{-\lambda(a-k)}\, d\lambda}$$

where $k = \sum_{i=1}^{n}(1 - e^{t_i})$

Then using the transformation $y = \lambda(a-k)$ and $\lambda = y/(a-k)$ and the derivative with respect to y is $d\lambda = dy/(a-k)$ then we obtain the final formula as:

$$\pi_2(\lambda \mid \underline{t}) = \frac{(a-k)^{n+1}\, \lambda^n\, e^{-\lambda(a-k)}}{\Gamma(n+1)} \quad \ldots (13)$$

Similarly, the third prior distribution is assumed to be the chi-squared distribution with hyper-parameter 'c'. The pdf of this prior is [17]:

$$g_3(\lambda) = \frac{\lambda^{(c/2)-1}\, e^{-\lambda/2}}{2^{c/2}\, \Gamma(c/2)} \quad ; \lambda > 0 \text{ and } c > 0 \quad \ldots (14)$$

The posterior distribution of the unknown parameter λ of BG have been obtain by combining eq. (5) and eq.(9) with eq.(14) as:

$$\pi_3(\lambda \mid \underline{t}) = \frac{\lambda^n e^{[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1-e^{t_i})]}\, \lambda^{(c/2)-1} e^{-\lambda/2}/2^{c/2}\, \Gamma(c/2)}{\int_0^{\infty} \lambda^n e^{[\sum_{i=1}^{n} t_i + \lambda \sum_{i=1}^{n}(1-e^{t_i})]}\, \lambda^{(c/2)-1} e^{-\lambda/2}/2^{c/2}\, \Gamma(c/2)\, d\lambda}$$

$$= \frac{\lambda^{n+(c/2)-1}\, e^{-\lambda(0.5-k)}}{\int_0^{\infty} \lambda^{n+(c/2)-1}\, e^{-\lambda(0.5-k)}\, d\lambda}$$

where $k = \sum_{i=1}^{n}(1 - e^{t_i})$

Then using the transformation $y = \lambda(0.5-k)$ and $\lambda = y/(0.5-k)$ and the derivative with respect to y is $d\lambda = dy/(0.5-k)$ then we obtain the final formula as:

$$\pi_3(\lambda \mid \underline{t}) = \frac{(0.5-k)^{n+(c/2)}\, \lambda^{n+(c/2)-1}\, e^{-\lambda(0.5-k)}}{\Gamma(n+(c/2))} \quad \ldots (15)$$

2.2. Gamma-Exponential and Chi-Square Distributions as Triple Priors

Consider the first prior distribution for λ as in eq.(10) and the second prior distribution for λ as in eq.(12) with the third prior distribution for λ as in eq.(14).

The triple prior distribution for λ can be defined by combining these three priors as follows:

$$g_4(\lambda) = \frac{\lambda^{a+(c/2)-2}\, e^{-\lambda(a+b+0.5)}\, a\, b^a}{2^{c/2}\,\Gamma(a+c/2)} \quad ; \lambda > 0 \text{ and } a, b, c > 0 \quad \ldots (16)$$

Hence, The posterior distribution of λ based on this triple prior distribution of λ for given data t can be obtained as follows:

$$\pi_4(\lambda \mid \underline{t}) = \frac{\lambda^n e^{[\sum_{i=1}^n t_i + \lambda \sum_{i=1}^n (1-e^{t_i})]}\, \frac{\lambda^{a+(c/2)-2} e^{-\lambda(a+b+0.5)}\, a\, b^a}{2^{c/2}\,\Gamma(a+c/2)}}{\int_0^\infty \lambda^n e^{[\sum_{i=1}^n t_i + \lambda \sum_{i=1}^n (1-e^{t_i})]}\, \frac{\lambda^{a+(c/2)-2} e^{-\lambda(a+b+0.5)}\, a\, b^a}{2^{c/2}\,\Gamma(a+c/2)}\, d\lambda}$$

$$= \frac{\lambda^{n+a+(c/2)-2}\; e^{-\lambda(a+b+0.5-k)}}{\int_0^\infty \lambda^{n+a+(c/2)-2}\; e^{-\lambda(a+b+0.5-k)}\, d\lambda}$$

where $k = \sum_{i=1}^n (1 - e^{t_i})$

By using the transformation y = λ (a+b+0.5 – k) and λ = y / (a+b+ 0.5 – k) and the derivative with respect to y is dλ = dy / (a+b+ 0.5 – k) then we obtain the final formula as:

$$\pi_4(\lambda \mid \underline{t}) = \frac{(a+b+0.5-k)^{n+a+(c/2)-1}\; \lambda^{n+a+(c/2)-2}\; e^{-\lambda(a+b+0.5-k)}}{\Gamma(n+a+(c/2)-1)} \quad \ldots (17)$$

2.3. Bayes Estimators Under the De-groot Loos Function(DLF) (Weighted Balance Loos Function):

In Bayesian estimation, we consider a type of loss function, which classified as a symmetric function was introduced by De-groot (2005) [16, 18, 19]:

$$L(\lambda, \hat{\lambda}) = \frac{(\lambda - \hat{\lambda})}{\hat{\lambda}^2} \quad \ldots (18)$$

$$\text{Risk} = E[L(\lambda, \hat{\lambda})] = E\,\frac{\lambda^2}{\hat{\lambda}^2} - \frac{2E(\lambda)}{\hat{\lambda}^2} + 1$$

Then

$$\frac{\partial}{\partial \hat{\lambda}}\left[E\,\frac{\lambda^2}{\hat{\lambda}^2} - \frac{2E(\lambda)}{\hat{\lambda}^2} + 1\right] = \frac{-2E(\lambda^2) + 2\hat{\lambda}\,E(\lambda)}{\hat{\lambda}^3}$$

The Bayes estimator under this a symmetric loss function is denoted by $\hat{\lambda}_d$:

$$\hat{\lambda}_d = \frac{E(\lambda^2 \mid \underline{t})}{E(\lambda \mid \underline{t})}$$

Therefor the Bayes estimators of λ based on the DLF is:

$$\hat{h}(t) = \frac{E(h^2(t) \mid \underline{t})}{E(h(t) \mid \underline{t})} \quad \ldots (19)$$

where

$$E(h(t) \mid \underline{t}) = \int_0^\infty h(t)\, \pi(\lambda \mid \underline{t})\, d\lambda$$

Now, the Bayes estimator of the hazard function h(t) corresponding to $\pi_1(\lambda \mid \underline{t})$ can be found as:

$$E_{\pi_1}(h(t) \mid \underline{t}) = \int_\lambda h(t)\, \pi_1(\lambda \mid \underline{t})\, d\lambda$$

$$= \int_0^\infty \lambda\, e^t\, \frac{(b-k)^{n+a}\, \lambda^{n+a-1}\, e^{-\lambda(b-k)}}{\Gamma(n+a)}\, d\lambda$$

$$= \frac{e^t\,(b-k)^{n+a}}{\Gamma(n+a)} \int_0^\infty \lambda^{n+a}\, e^{-\lambda(b-k)}\, d\lambda$$

By using the transformation y = λ (b – k) which implies that λ = y /(b – k) and the derivative with respect to y is dλ = dy / (b – k):

$$E_{\pi_1}(h(t) \mid \underline{t}) = \frac{e^t\,\Gamma(n+a+1)}{(b-k)\,\Gamma(n+a)} \quad \ldots (20)$$

Now

$$E_{\pi_1}(h^2(t) \mid \underline{t}) = \int_\lambda h^2(t)\, \pi_1(\lambda \mid \underline{t})\, d\lambda$$

$$= \int_0^\infty \lambda^2\, e^{2t}\, \frac{(b-k)^{n+a}\, \lambda^{n+a-1}\, e^{-\lambda(b-k)}}{\Gamma(n+a)}\, d\lambda$$

$$= \frac{e^{2t}\,(b-k)^{n+a}}{\Gamma(n+a)} \int_0^\infty \lambda^{n+a+1}\, e^{-\lambda(b-k)}\, d\lambda$$

Then

$$E_{\pi_1}(h^2(t) \mid \underline{t}) = \frac{e^{2t}\,\Gamma(n+a+2)}{(b-k)^2\,\Gamma(n+a)} \quad \ldots (21)$$

Substituting eq. (20) and eq.(21) in eq.(19) we have:

$$\hat{h}_1(t) = \frac{e^{2t}\,\Gamma(n+a+2)\,(b-k)\,\Gamma(n+a)}{(b-k)^2\,\Gamma(n+a)\,e^t\,\Gamma(n+a+1)} = \frac{e^t\,(n+a+2)}{(b-k)} \quad \ldots (22)$$

The Bayes estimator of the hazard function h(t) corresponding to $\pi_2(\lambda \mid \underline{t})$, $\pi_3(\lambda \mid \underline{t})$ and $\pi_4(\lambda \mid \underline{t})$ can be found by similarly method and we get the following formulas:

$$\hat{h}_2(t) = \frac{e^t\,(n+3)}{(a-k)} \quad \ldots (23)$$

$$\hat{h}_3(t) = \frac{e^t\,(n+(c/2)+2)}{(0.5-k)} \quad \ldots (24)$$

$$\hat{h}_4(t) = \frac{e^t\,(n+a+(c/2)+1)}{(b+a+0.5-k)} \quad \ldots (25)$$

2.4. Estimate Failure Rate Function Using Numerical Method

In this section, we introduce expansion method by two ways (Bernstein polynomial and power function), which is use to estimate the failure rate function h(t), as follows [11, 12]:

$$h_n(t) = \sum_{m=0}^n c_m P_m(t) \qquad t \ge 0 \quad \ldots (26)$$

where cm are unknown coefficients and Pm(t) are known functions.

The first way, we take $h_n(t) = h_B(t)$ and $P_m(t) = B_m^n(t)$, where $B_m^n(t)$ are Bernstein polynomials which are given by [8]:

$$B_m^n(t) = \binom{n}{m} (1-t)^{n-m}\, t^m \qquad m = 0, 1, \dots, n \quad ,$$
$$0 \le t \le 1 \qquad \dots (27)$$

and n is the polynomial degree.

The second way, we take $h_n(t) = h_P(t)$ and $P_m(t)$ a set of power functions as:

$$P_m(t) = t^m \qquad m = 0, 1, \dots, n \qquad \dots (28)$$

Now, let $\{ t_0, \dots, t_n \}$ be a set of points in the subinterval $[t_0, t_n]$, that is equation(26) becomes:

$$h_n(t_k) = \sum_{m=0}^{n} c_m P_m(t_k) \qquad k = 0, \dots, n \qquad \dots (29)$$

Recall equation(8), and substitute $\hat{h}_{ML}(t)$ into equation(29) we have:

$$\sum_{m=0}^{n} c_m P_m(t_k) = \hat{\lambda}_{ML} e^{t_k} \qquad k = 0, \dots, n$$

From above equation, we are obtain two linear systems of (n) equations and (n) unknowns c_m , m= 0, …, n. The first system, where Pm(t) as equation (27) and the second as equation (28).

Finally, solve these systems for coefficients c_m's using Gauss-elimination to find the approximate solution of h(t) by two ways.

When using Bernstein polynomials, we need to transform the interval [a, b] to [0, 1], then we can convert the variable so that the problem is reformulated on [0, 1], as following: where x ϵ [a, b] let t = (x- a)/(b-a) then

$$B_m^n\left(\frac{x-a}{b-a}\right) = \binom{n}{m} \left(1 - \frac{x-a}{b-a}\right)^{n-m} \left(\frac{x-a}{b-a}\right)^m \qquad m = 0, 1, \dots, n$$

2.5. Examples

Test examples are present in this section for different values (n) of different intervals to find best estimate of failure rate function h(t) by using mean square error, show in tables (1- 5). Where $\hat{\lambda}_{ML}$, k as in equation(6) and t ϵ $[t_1, t_n]$ with $t_i = t_1 + (i-1)h$, i= 1,2, …, n and $h = (t_n - t_1)/n$.

Table 1: Example1 with [0, 1]

err \ n		n=10		n=25		n=50	
$err_B=\sum(h(t)-h_B(t))^2$		1.4791e-30		1.0502e-29		3.0596e-27	
$err_P=\sum(h(t)-h_P(t))^2$		1.5777e-30		1.5185e-25		4.3769e-06	
$err_1=\sum(h(t)-\hat{h}_1(t))^2$	a	1.8055e-08	0.998	7.4971e-09	0.777	8.8454e-08	0.782
	b		1.9		1.9		1.95
$err_2=\sum(h(t)-\hat{h}_2(t))^2$	a	1.6780e-07	1.901	8.4856e-08	2.052	2.7753e-06	2.1
$err_3=\sum(h(t)-\hat{h}_3(t))^2$	c	7.0471e-09	-2.422	6.4236e-09	-2.538	5.9343e-05	-2.53
$err_4=\sum(h(t)-\hat{h}_4(t))^2$	a	1.4333e-09	0.1	9.7818e-12	0.274	5.5507e-11	0.349
	b		0.1		0.1		0.1
	c		0.009		0.007		0.009

Table 2: Example2 with [0, 10]

err \ n		n=10		n=25		n=50	
$err_B=\sum(h(t)-h_B(t))^2$		1.9983e-37		1.1520e-36		1.6234e-35	
$err_P=\sum(h(t)-h_P(t))^2$		4.7356e-26		1.4302e-29		3.2802e-12	
$err_1=\sum(h(t)-\hat{h}_1(t))^2$	a	2.7166e-08	0.007	1.0391e-08	0.001	4.8349e-10	0.01
	b		2571		3582		3397
$err_2=\sum(h(t)-\hat{h}_2(t))^2$	a	2.2081e-05	3841	2.0440e-11	5371	2.8896e-10	5966
$err_3=\sum(h(t)-\hat{h}_3(t))^2$	c	7.0533e-08	-4	9.6271e-09	-3.999	6.1649e-09	-3.999
$err_4=\sum(h(t)-\hat{h}_4(t))^2$	a	2.7310e-10	0.35	9.9996e-11	0.35	2.7784e-12	0.35
	b		1734		2425		2703
	c		0.009		0.01		0.0193

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Table 3: Example3 with [0, 20]

err \ n		n=10		n=25		n=50	
err$_B$=$\sum(h(t)-h_B(t))^2$		1.1911e-44		1.7691e-44		6.2183e-44	
err$_P$=$\sum(h(t)-h_P(t))^2$		3.5705e-22		1.4379e-18		4.8257e-16	
err$_1$=$\sum(h(t)-\hat{h}_1(t))^2$	a	1.2412e-16	0.0001	1.4125e-16	0.0001	3.6861e-18	0.0001
	b		15188140		31671854		39460355
err$_2$=$\sum(h(t)-\hat{h}_2(t))^2$	a	4.0640e-16	22781071	6.0777e-17	47505406	6.1304e-20	59187573
err$_3$=$\sum(h(t)-\hat{h}_3(t))^2$	c	3.3019e-15	-4	3.7881e-16	-4	1.2677e-16	-4
err$_4$=$\sum(h(t)-\hat{h}_4(t))^2$	a	9.2855e-16	0.001	2.1282e-16	0.001	3.8649e-17	0.001
	b		7605080		15858888		19758784
	c		0.001		0.001		0.001

Table 4: Example4 with [2, 5]

err \ n		n=10		n=25		n=50	
err$_B$=$\sum(h(t)-h_B(t))^2$		2.5278e-34		2.0102e-33		1.0022e-30	
err$_P$=$\sum(h(t)-h_P(t))^2$		1.6027e-29		1.8321e-26		2.4267e-17	
err$_1$=$\sum(h(t)-\hat{h}_1(t))^2$	a	2.5398e-12	0.5694	1.7072e-12	0.7287	1.4648e-13	0.57779
	b		101		118		115
err$_2$=$\sum(h(t)-\hat{h}_2(t))^2$	a	3.5688e-07	118	2.1235e-06	130	4.1813e-07	134
err$_3$=$\sum(h(t)-\hat{h}_3(t))^2$	c	1.5758e-10	-3.9745	1.0495e-11	-3.9769	2.0919e-12	-3.9776
err$_4$=$\sum(h(t)-\hat{h}_4(t))^2$	a	3.0575e-11	0.0009	2.1384e-12	0.001	1.0935e-12	0.009
	b		39		43		48
	c		0.008		0.0099		0.1567

Table 5: Example5 with [5, 10]

err \ n		n=10		n=25		n=50	
err$_B$=$\sum(h(t)-h_B(t))^2$		4.9959e-38		2.1453e-37		9.7654e-36	
err$_P$=$\sum(h(t)-h_P(t))^2$		1.3982e-23		5.8073e-28		3.7265e-22	
err$_1$=$\sum(h(t)-\hat{h}_1(t))^2$	a	2.2123e-12	0.0009	1.3667e-12	0.00981	2.1667e-13	0.00939
	b		6746		7942		8358
err$_2$=$\sum(h(t)-\hat{h}_2(t))^2$	a	2.8152e-09	10114	8.0127e-11	11855	4.7562e-10	12478
err$_3$=$\sum(h(t)-\hat{h}_3(t))^2$	c	7.1577e-13	-3.9997	2.3666e-13	-3.99975	2.1803e-15	-3.99976
err$_4$=$\sum(h(t)-\hat{h}_4(t))^2$	a	3.9448e-12	0.0001	3.5460e-13	0.00099	1.0183e-13	0.00096
	b		3388		3957		4165
	c		0.0099		0.00099		0.00098

3. CONCLUSINS

The most important conclusion for estimating the shape parameter (λ) of basic Gompertz distribution, with the assumption that the scale parameter is known are:

1) From tables (1), (2) and (3) of the intervals [0, 1], [0, 10] and [0, 20] respectively, the results show that the value of (MSE) of all estimator are decreasing with increasing interval for all sample size.
2) From all tables (1-5) the numerical method by using Bernstein polynomials gives best estimate for all different value of (n).

3) From table (5) the results show the values of (MSE) for $\hat{h}_3$ is less than $\hat{h}_1$, $\hat{h}_2$ and $\hat{h}_4$ for all different value of (n).
4) In general, from the numerical methods, we can notice that (Bernstein polynomials method) takes the best results from the (power function) for all different value of (n) and different intervals, but Bernstein polynomials method requires converting all intervals into [0, 1], therefore as future research, some suggestions have been put forward: using another distribution define of interval [0, 1].
5) From table (3) of the interval [0, 20] and table (4) of the interval [2, 5] show the values of (MSE) for $\hat{h}_2$ and $\hat{h}_1$ respectively are the best estimation for all different values of (n).

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الطرق العددية للتوزيع الإولي الثلاثي المعلوم لمعدل الفشل لنموذج بيسك جومبيرتز

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الخلاصة:

في هذا البحث تم التعامل مع توزيع Basic Gompertz. واستخدمت طريقة الامكان الاعظم وطريقة بيز لتقدير معلمة الشكل غير المعلومة. تم الحصول على دالة معدل الفشل باستخدام دالة الخسارة المتماثلة (Degroot loss function) ودوال التوزيع الاولي المختلفة (كاما ، الأسي ، مربع كاي ودالة التوزيع الاولي الثلاثية)، واجريت مقارنة حول أداء هذه المقدرات مع الحل العددي الذي وجد باستخدام طرق التوسيع (Bernstein polynomial and power function) والتي طبقت لايجاد دالة معدل الفشل عددياً. تم إجراء اختبار الكفاءة للطرق المقترحة مع عدد من أمثلة الاختبار. وأخيراً، تم استخدام Matlab (R2015b) للحسابات.