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Depth estimation of spherical bodies using Differential Operators (Gradient $\Pi\vec{g}\Pi$, Laplacian $\nabla^2 Z$ and Biharmonic $\nabla^4 Z$) to its gravity fields.

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ABSTRACT

Differential Operators (Gradient, Laplacian and Biharmonic) have been used to determine anomaly characteristics using theoretical gravity field for spherical bodies with different depths, radius and density contrasts. The intersection between the gravity field and the three differential operator's fields could be used to estimate the depth to the center of the spherical bodies regardless their different radius, depths and density contrasts. The Biharmonic Operator has an excellent result, were two zero closed contours lines produced. The diameter of the internal closed zero contour line define almost precisely the depth to the center of spherical bodies. This is an attempt to use such technique to estimate depths. Also, the Biharmonic Operator has very sensitivity to resolve hidden small anomaly due the effect of large neighborhood anomaly, the 2nd derivative Laplacian Filter could reveal these small anomaly but the Biharmonic Operator could indicate the exact depth. The user for such technique should be very care to the accuracy of digitizing the data due to the high sensitivity of Biharmonic Operator.The validity of the method is tested on field example for salt dome in United States and gives a reasonable depth result.

Introduction:

In gravity data interpretations, finding depth to center of mass and/or the top of the body causing the anomaly is of major important. The maximum depth at which the top of any particular geological body can be situated is known as the limiting depth. Methods of obtaining this information depend on which interpretational technique and model are being used (1).

Another major important is to delineate the edge of the buried objects. The detection of border of subsurface bodies can be investigated by using either derivative based classical approaches or contemporary image processing algorithms (2).

Simple geometrically shaped models can be very useful in quantitative interpretation of gravity data acquired in a small area over the buried structure. The models may not be geologically realistic, but usually approximate equivalence is sufficient to determine whether the form and magnitude of calculated gravity data to make the geological postulate reasonable (3).

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The interpreter normally use simple geometrical shape models such as sphere, horizontal or vertical cylinder, dyke, prisms and contact (fault) and calculate their theoretical gravity effects to find any rules that could help him to know the depth directly from a profile measurements. For a spherical shape body, the half-width (X1/2) method is the commonest rules of thumb, these named Smith Rules (4).

Several numerical methods have been developed by various authors for interpreting gravity anomalies caused by simple models to find the depth of most geological structures. Excellent reviews are given by (3). Nabighian et al. (5) present excellent historical reviews for the development of the gravity method in exploration. Their paper includes the main developments in gravity instrumentation, data reduction and processing, data filtering, enhancement with data interpretation. Also, they summarizes a timeline of gravity exploration include the date and important event type.

For the first time, the present paper is aiming to use a method to estimate the depth to the center of three dimensional spherical bodies by applying Differential Operators (Gradient $\Pi\vec{g}\Pi$, Laplacian $\nabla^2 Z$ and

Biharmonic $\nabla^4 Z$) to its gravity fields.

THEORETICAL BACK GROUND:

The branch of mathematics that deals with derivatives is called Differential Calculus (6). Famous contouring program - Surfer Program (Version 7.0 and later) (7) can calculate the Differential Operator for a grid data. The Differential Operator includes Gradient Operator, Laplacian Operator, and Biharmonic Operator.

Gradient Operator: generates a grid of steepest slopes (i.e. the magnitude of the gradient) at any point on the surface (8). The Gradient Operator is zero for a horizontal surface, and approaches infinity as the slope approaches vertical. The definition of the gradient yields the following equation:

$$\Pi\vec{g}\Pi = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$$

Laplacian Operator: provides a measure of discharge or recharge on a surface (7 and 8). In grid files generated with the Laplacian Operator, recharge areas are positive, and discharge areas are negative. Groundwater, heat, and electrical charge are three examples of conservative physical quantities whose local flow rate is proportional to the local gradient. The Laplacian operator, $\nabla^2 Z$ is the mathematical tool that quantifies the net flow into (Laplacian > 0, or areas of recharge) or out of (Laplacian < 0, areas of discharge) a local control volume in such physical situations. The Laplacian Operator is defined in multivariable calculus by:

$$\nabla^2 Z = \frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2}$$

In Image Processing the Laplacian responds to transitions in intensity, it is seldom used in practice for edge detection. As a second-order derivative, the Laplacian typically is unacceptably sensitive to noise, Moreover, the Laplacian produces double edges and is unable to detect edge direction (9).

Biharmonic Operator: Bending of thin plates and shells, viscous flow in porous media, and stress functions in linear elasticity are three examples of physical quantities that can be mathematically described Biharmonic Operator (7 and 10). The Biharmonic Operator, $\nabla^4 Z$, is defined in multivariable calculus by:

$$\nabla^4 Z = \frac{\partial^4 Z}{\partial x^4} + 2\frac{\partial^4 Z}{\partial x^2 \partial y^2} + \frac{\partial^4 Z}{\partial y^4}$$

This is comparable to applying the Laplacian Operator twice (bilaplacian).

METHODOLOGY:

To apply the Differential Operator on simple geometrical shape, Sphere Subroutine (written in FORTRAN and published in (11)) has been used and applied to nine spheres with different radius, depths and density contrasts. Table (1) shows the data used for each sphere. For each sphere, the theoretical gravity field has been calculated for three depths 10, 15 and 20 km. The total cases are 27. The dimension of the models is 64x64 km. Figures 1, 2 and 3 represent the 27 case mentioned above.

After that, Surfer 9.0 program used to apply the Differential Operator for all these 27 case. Figure (4) illustrate 2D and 3D representation for sphere No.1 with radius 2.5 km, depth 10 km and density contrast 0.2 g/cc. Then, a slice profile across the center of the anomalies has been taken and the four curves (Gravity, Gradient, Laplacian and Biharmonic) are plotted on one graph to be apple to make comparison between them as shown in figure (5). Its clear from Fig.(5) that the intersection between the gravity field and the three differential operator's fields could be used to estimate the depth to the center of the spherical body. The locations of the intersection between gravity field curve and Laplacian curve with the maxima of the gradient profile with almost nearly zero line for Biharmonic curve are determining the exact depth to the center of the spherical model. Also, the location of the intersection between the Gradient and Biharmonic curves define the half depth to the center of the sphere. The same approaches have been done for all spherical models and give the same results regardless its differences in radius, depths and density contracts. Figures (6 and 7) shows an example for the same sphere (Radius 2.5 km, Density Contrast 0.2 g/cc but with depth 15 and 20 km to its center) and give the same results.

Return back to figure (4), its clear that the Biharmonic Operator map has a Mexican hat shape with two zero closed contours; the internal closed zero contour line define almost precisely the depth to the center of spherical bodies (Figure, 8). These characteristic shape and width of the internal zero contour have been tested for all 27 case; the Biharmonic Operator gives an excellent results and the exact depth could be measured directly from Biharmonic Operator maps. Figures (9 and 10) are example of such process for different spheres.

COMPLICATE THE TEST:

In potential fields' survey, the observed data comprise the sum of the effects produced by all underground sources. The targets are often small-scale structures buried at shallow depths. The response of these targets is superimposed in a regional field which arises from underground sources that are usually larger in size or buried deeper. Trying to test the procedure on complicated models, a case for three spheres with different depths, radius and density contrasts habe been used. Figure (11) show 2D and 3D presentation for this model. From the gravity map in the figure, it's very difficult to recognize the small sphere (sphere no. 2 in the model) due to the effect of sphere no. 3 that has larger size and density contrast.

The second derivative operator could resolve these anomalies (applying a simple 3x3 Laplacian filter which has the following coefficients (7)):

0	-1	0
-1	4	-1
0	-1	0

The Laplacian filter produces a curvature map in which inflection points in the original data are located at the zero contours Figure (12). These procedures are widely used in image processing technique. But, the Biharmonic Operator could resolve the anomaly for sphere no. 2 and calculate its depth accurately.

FIELD EXAMPLE:

To estimate the applicability and stability of the present method, Humble Salt Dome Anomaly is chosen as a field example (12). The Bouguer gravity map of the Humble Salt Dome near Houston is given in Figure (13). The average depth according to (12 and 3) to the center of the salt dome is 4.92 km. But no one gives the estimated boundary of this dome.

Most authors avoid taking 3D field example to test their interpretation procedures. They take a profile across the centre of the anomaly and apply simplified interpretation tools to win reasonable results (as done in 12 and 3). These are due difficulties in finding typical field example for spherical body hypothesis.

The Differential Operators are very sensitive due its dealing with derivatives of different degrees, where the high frequency signals will amplified greatly due to this process. The different in data gradient also affects the result. White noise data form small error in digitization the original data also amplified by derivatives of different degrees. All these facts must be taken in consideration and a type of smoothing should be applied the data to enhance the signal-to-noise ratio. Many authors discuss these effects especially for method that use derivatives in depth estimation. Pašteka et al. (13) present most up-to-date summery for this problem and suggest a type of regularized filter to damp the amplification of the high frequency content in the processed signal.

For that, smoothing the input data is mandatory to get reasonable result. Figure (14 a) shows a smooth digitized gravity field for the Humble Dome. The dimension of the taken field example is nearly 16.8km X 14km with grid interval nearly 0.66km. Figure (14 b, c and d) is the output of Gradient, Laplacian and Biharmonic Operators. The Gradient map clearly shows that the Southeastern part of the anomaly has highest gradient. Matrix smooth with 1x1 cell is applied to the Biharmonic map Figure (14 d). The zero closed contour line for the Biharmonic map show that the direct calculation of the depth is differs from side to side. The scale bar on the figure could be used as a ruler for direct measuring to depth. The calculated depth to the center of the salt dome is less than 5 km in some direction and more than 5 km in NW direction. Actually, this zero closed contour illustrates the shape and trend of the Humble Salt Dome. More simplified way could be done by drawing a smooth circle around the zero closed contour (Fig. 14 d, the dash line circle) to get mean estimation for depth. This result is very close to the depth given by (12 and 3).

CONCLUSION:

For the first time, The Differential Operators are operated to the gravity field to estimate depth to the center of spherical bodies. The Biharmonic Operator is very sensitive to determine the shape and depth were the zero closed contour is the key factor for that. Smoothing should be applied to the real field data and Biharmonic Operator maps to enhance the signal-to-noise ratio and get an accurate result.

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Table (1) shows the 27 cases used to apply by Differential Operator.

Sphere No.1	Sphere No.2	Sphere No.3
Radius: 2.5 km Density Contrast: 0.2 g/cc Depth to the center: 10, 15 and 20 km	**Radius: 2.5 km Density Contrast: 0.3 g/cc Depth to the center: 10, 15 and 20 km**	**Radius: 2.5 km Density Contrast: 0.4 g/cc Depth to the center: 10, 15 and 20 km**
Sphere No.4	**Sphere No.5**	**Sphere No.6**
Radius: 5 km Density Contrast: 0.2 g/cc Depth to the center: 10, 15 and 20 km	**Radius: 5 km Density Contrast: 0.3 g/cc Depth to the center: 10, 15 and 20 km**	**Radius: 5 km Density Contrast: 0.4 g/cc Depth to the center: 10, 15 and 20 km**
Sphere No.7	**Sphere No.8**	**Sphere No.9**
Radius: 7.5 km Density Contrast: 0.2 g/cc Depth to the center: 10, 15 and 20 km	**Radius: 7.5 km Density Contrast: 0.3 g/cc Depth to the center: 10, 15 and 20 km**	**Radius: 7.5 km Density Contrast: 0.4 g/cc Depth to the center: 10, 15 and 20 km**

Sphere No.1

Sphere No.2

Sphere No.3

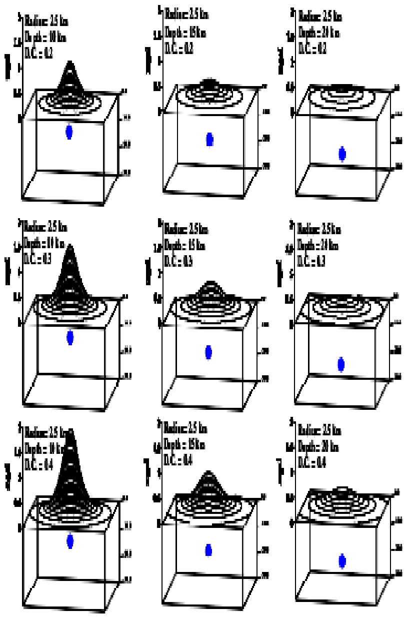

Fig. (1) 3D Representation for the gravity field for spheres No. 1, 2 and 3.

Sph ere No.4

Sph ere No.5

Sph ere No.6

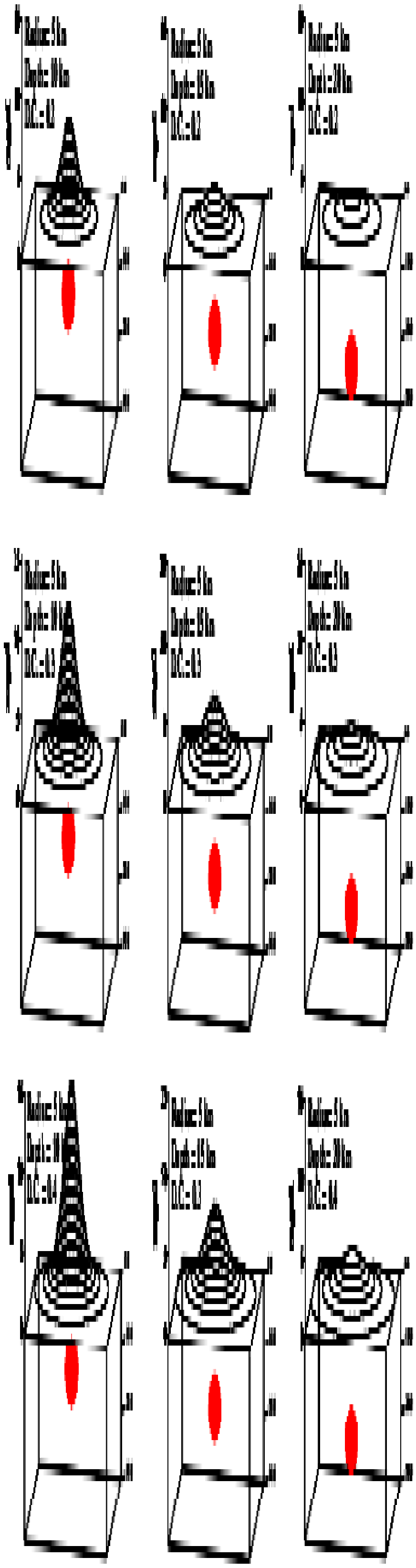

Fig. (2) 3D Representation for the gravity field for spheres No. 4, 5 and 6.

Sphere No.7

Sphere No.8

Sphere No.9

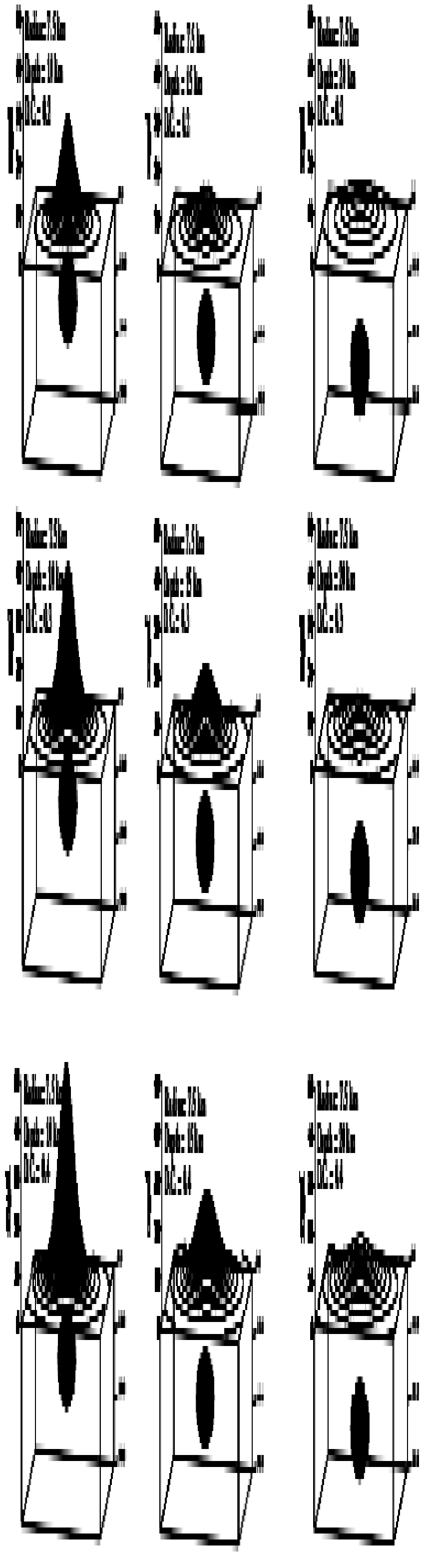

Fig. (3) 3D Representation for the gravity field for spheres No. 7, 8 and 9.

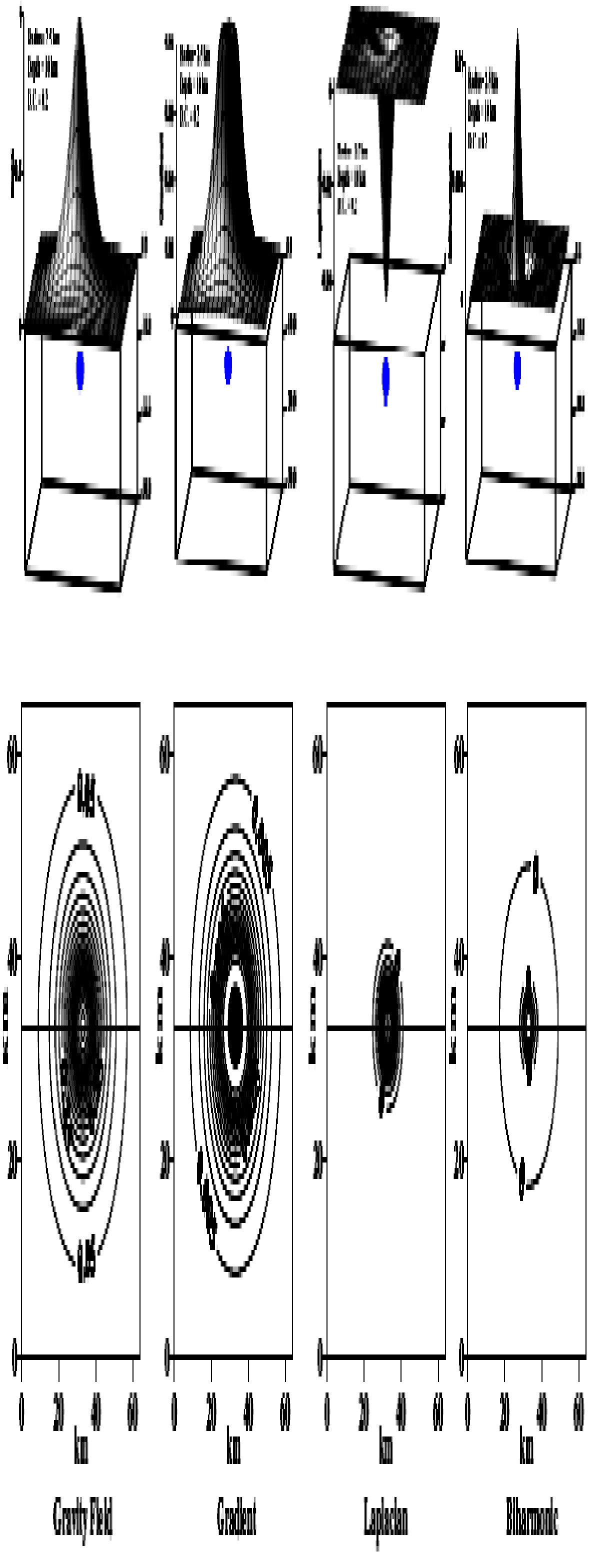

Fig. (4)3D and 2D presentation for spherical case with radius 2.5 km, depth 10 km and density contrast 0.2 g/cc. A profile taken across the middle part of each map (Gravity, Gradient, Laplacian and Biharmonic) and the results shown in Fig. (5).

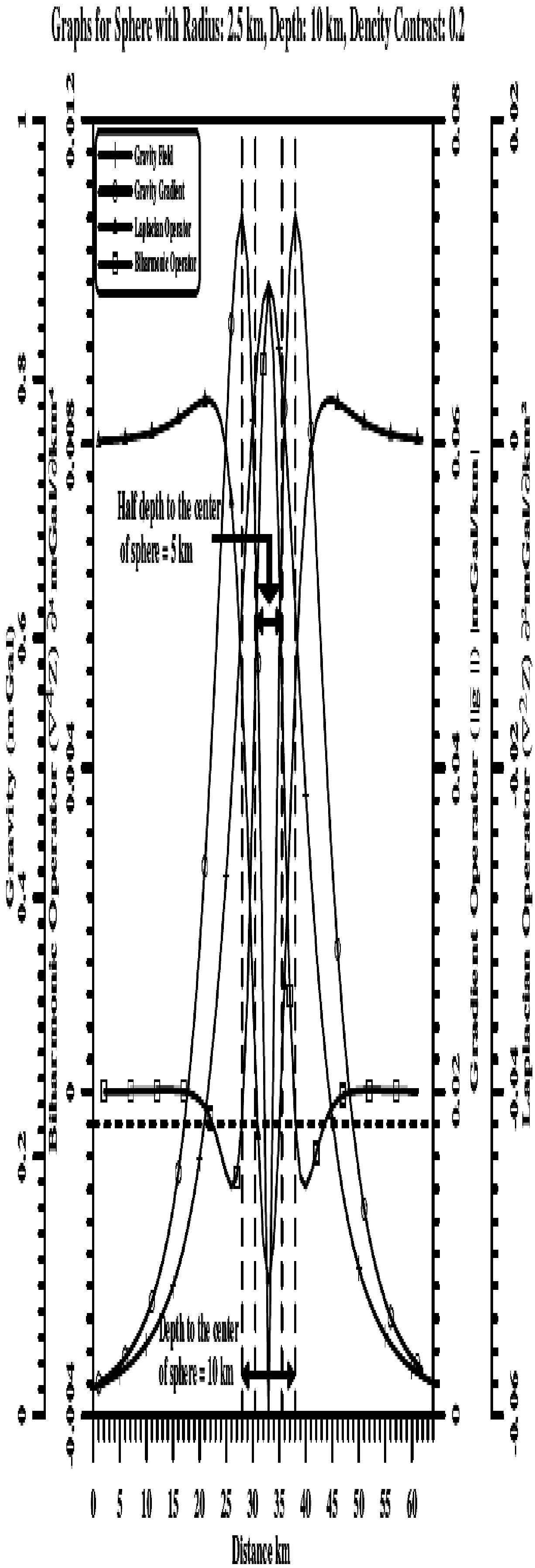

Fig. (5) Represent the profiles taken across the center of sphere shown in Fig. (4).

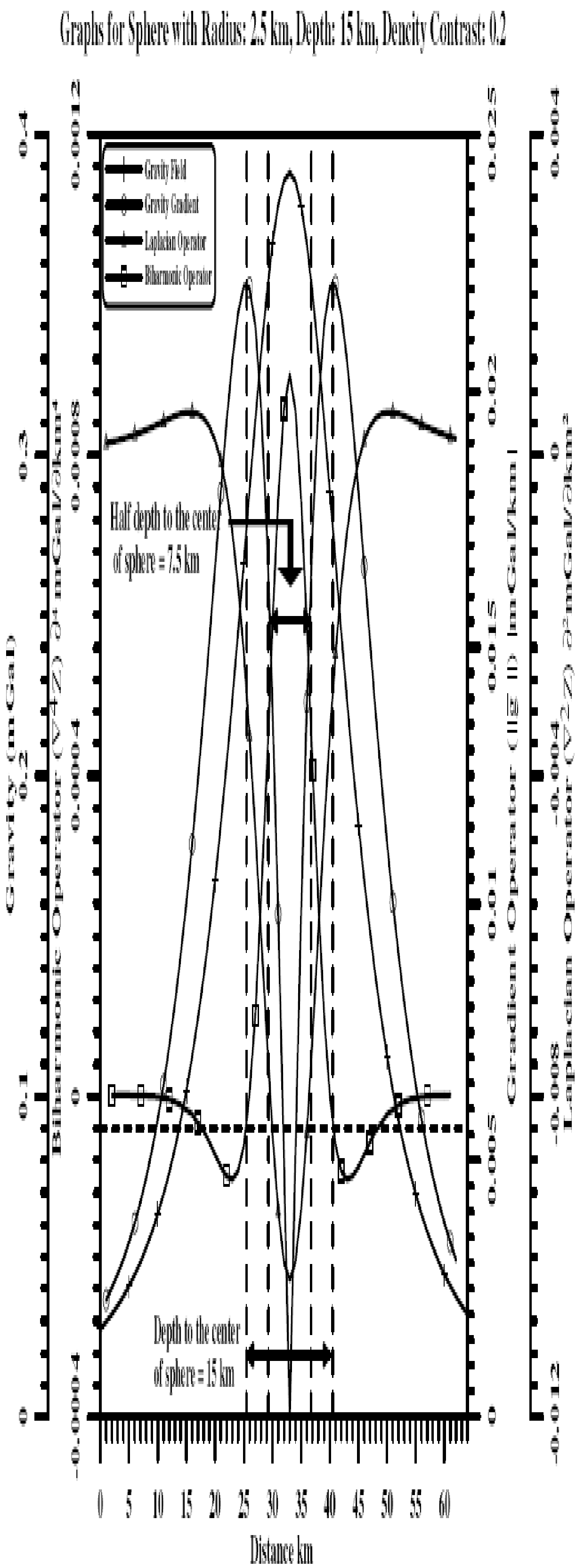

Fig. (6) Represent the profiles taken across the center of sphere with Radius 2.5 km, depth 15 km and Density Contrast 0.2 g/cc.

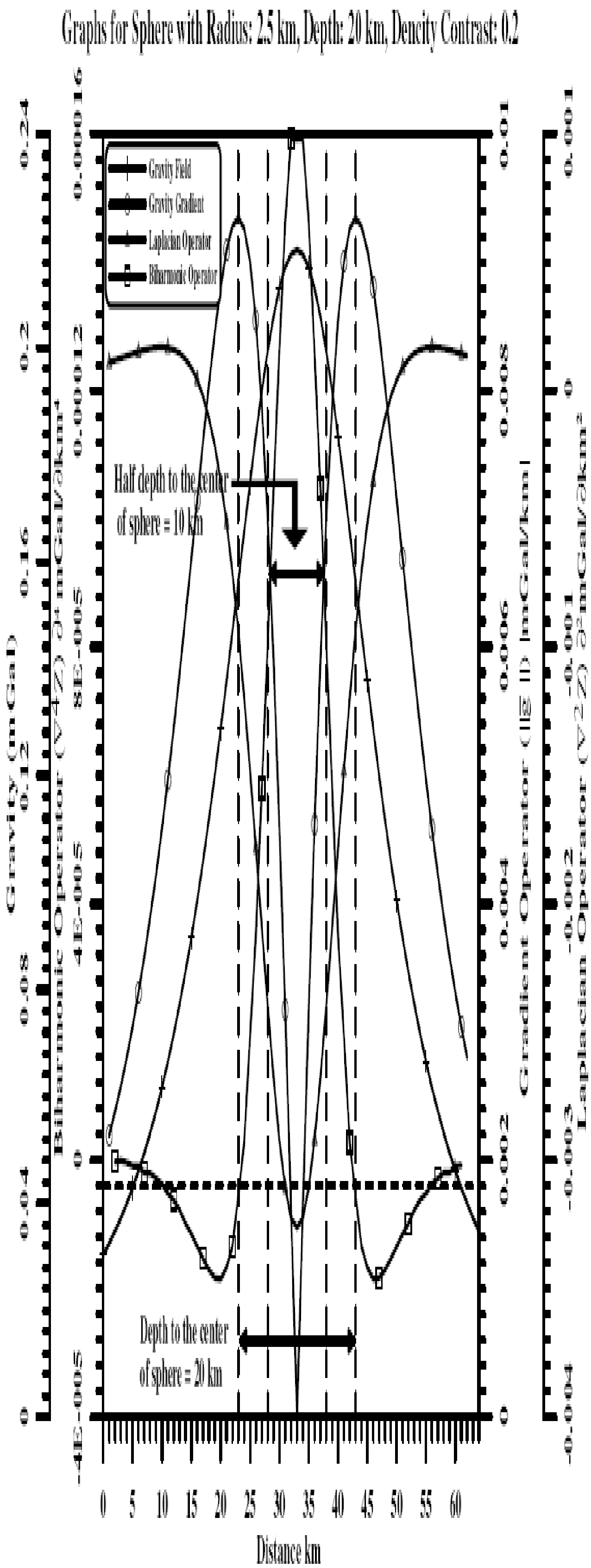

Fig. (7) Represent the profiles taken across the center of sphere with Radius 2.5 km, depth 20 km and Density Contrast 0.2 g/cc.

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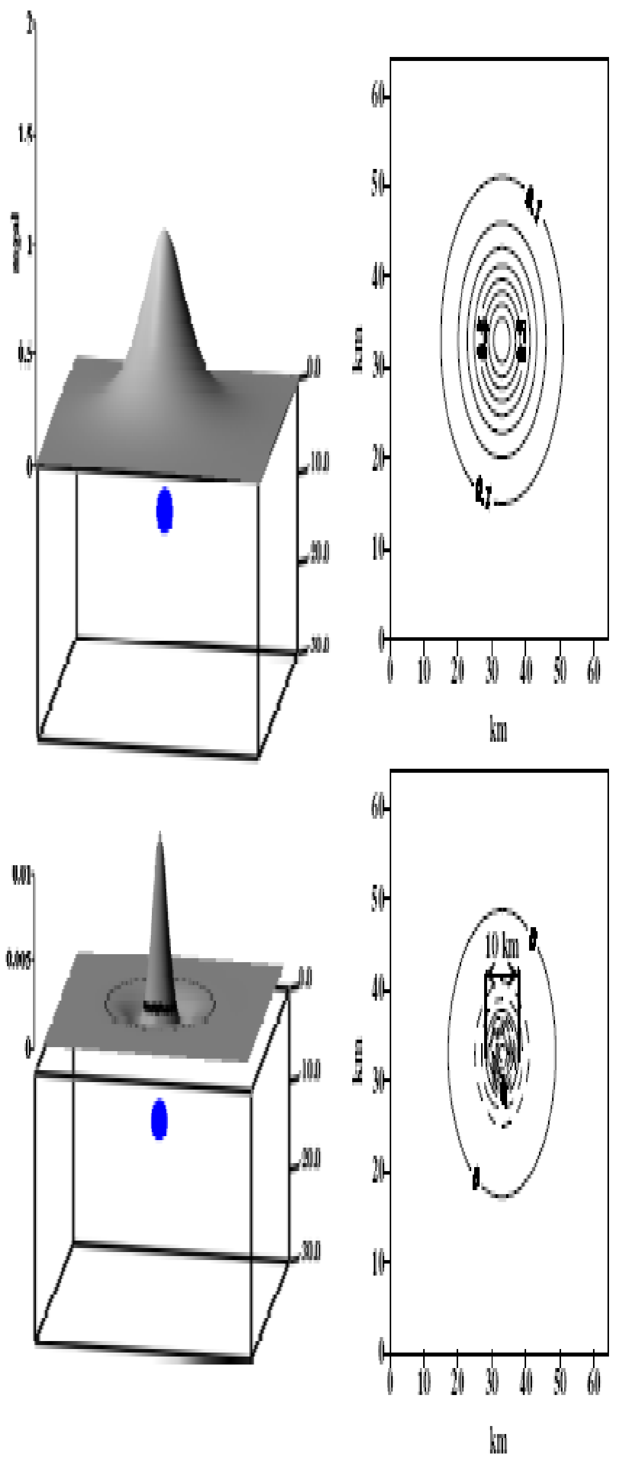

Fig. (8) Illustrate the Gravity and Biharmonic maps for sphere with radius 2.5 km, depth to the center 10 km and Density Contrast 0.2 g/cc. It is clear that, the depth could be calculated directly from the Biharmonic map that has a Mexican hat shape.

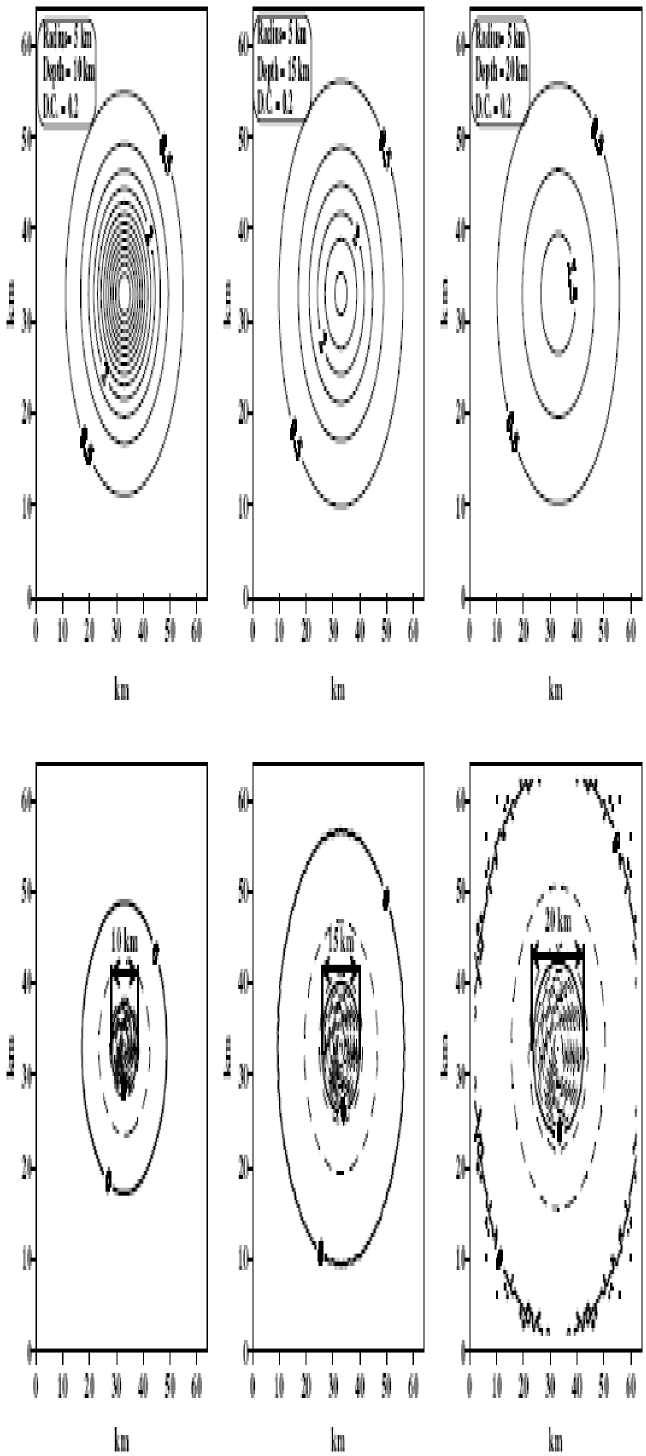

Fig. (9) Illustrate the Gravity and Biharmonic maps with Radius 5 km, Density contrast 0.2 g/cc but have different depths 10, 15 and 20 km. These depths could be calculated directly from the Biharmonic maps.

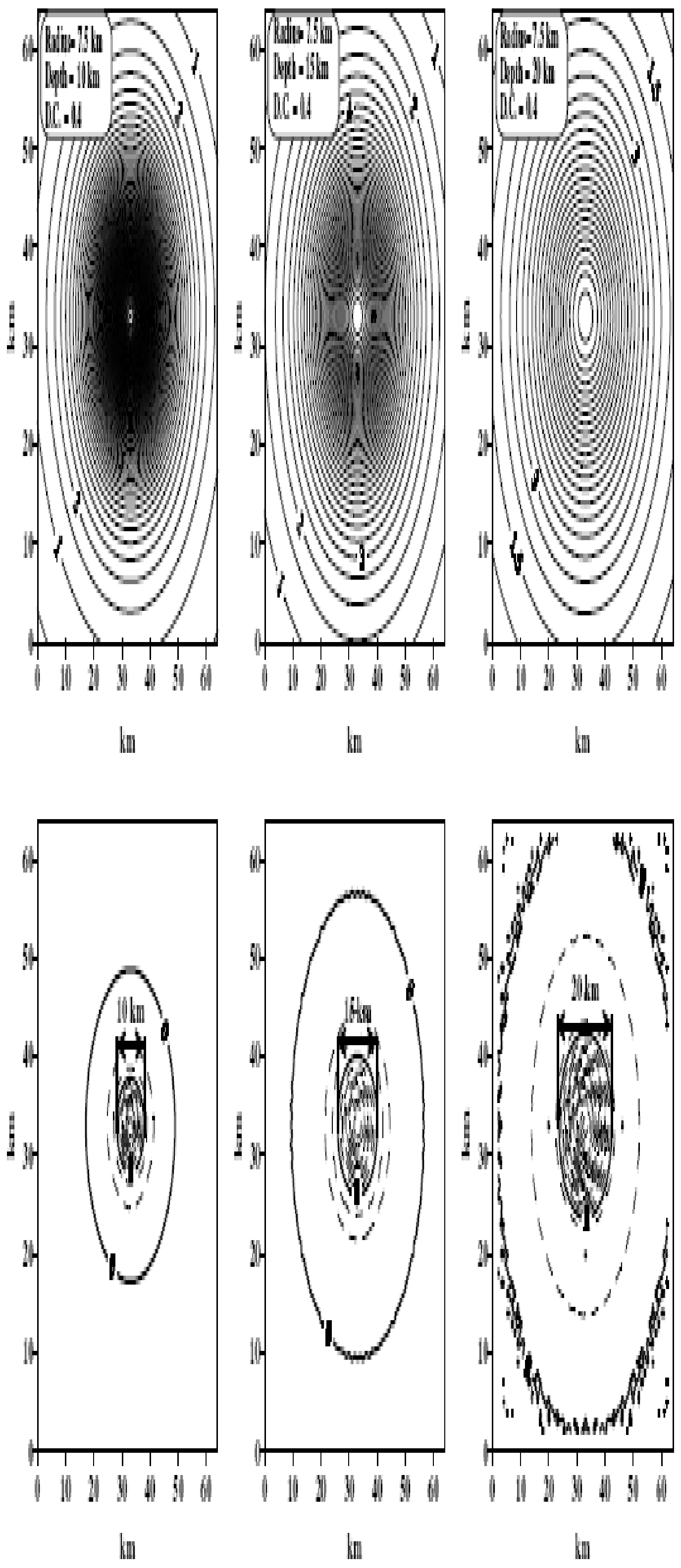

Fig. (10) Illustrate the Gravity and Biharmonic maps with Radius 7.5 km, Density contrast 0.2 g/cc but have different depths 10, 15 and 20 km. These depths could be calculated directly from the Biharmonic maps.

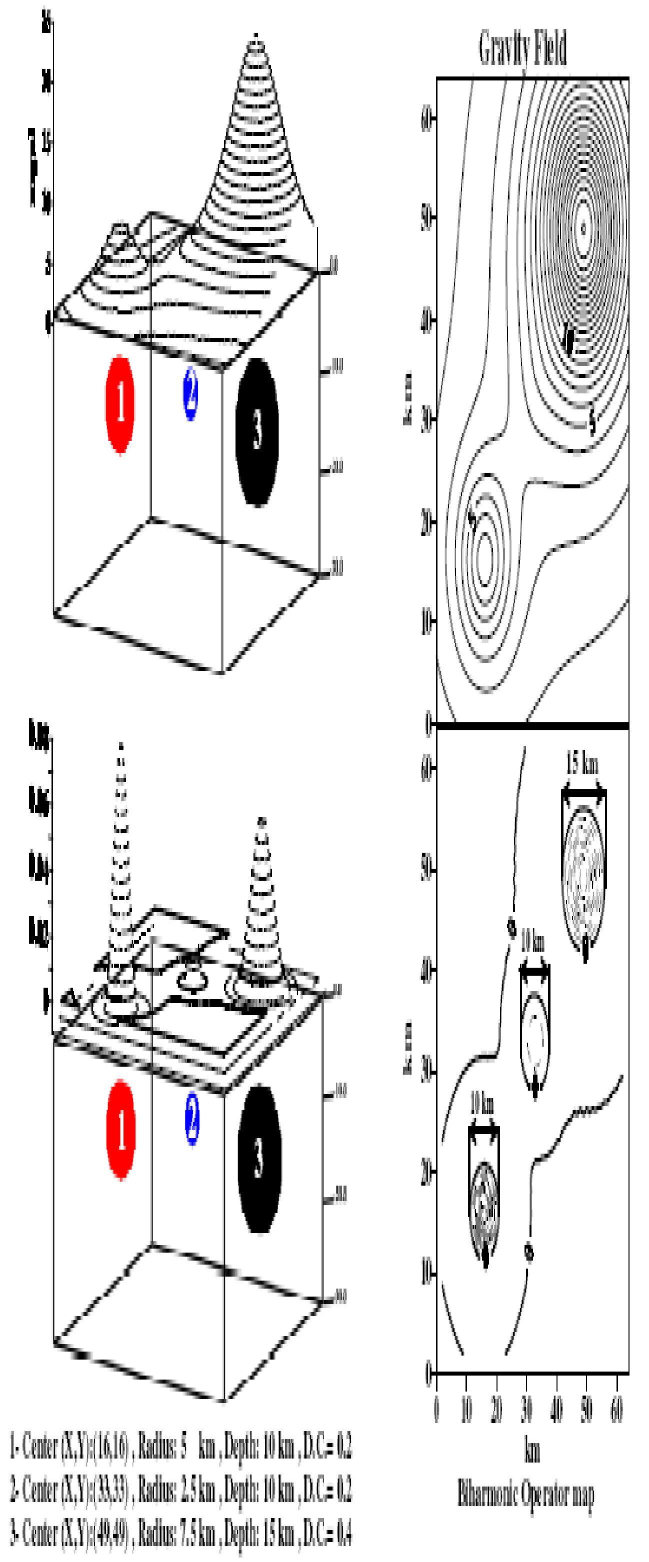

Fig. (11) Show 2D and 3D presentation of a complicated model. The parameters for the calculated spheres are shown in the figure. The dimension of the model is 64X64 km with 2.5 km grid spacing.

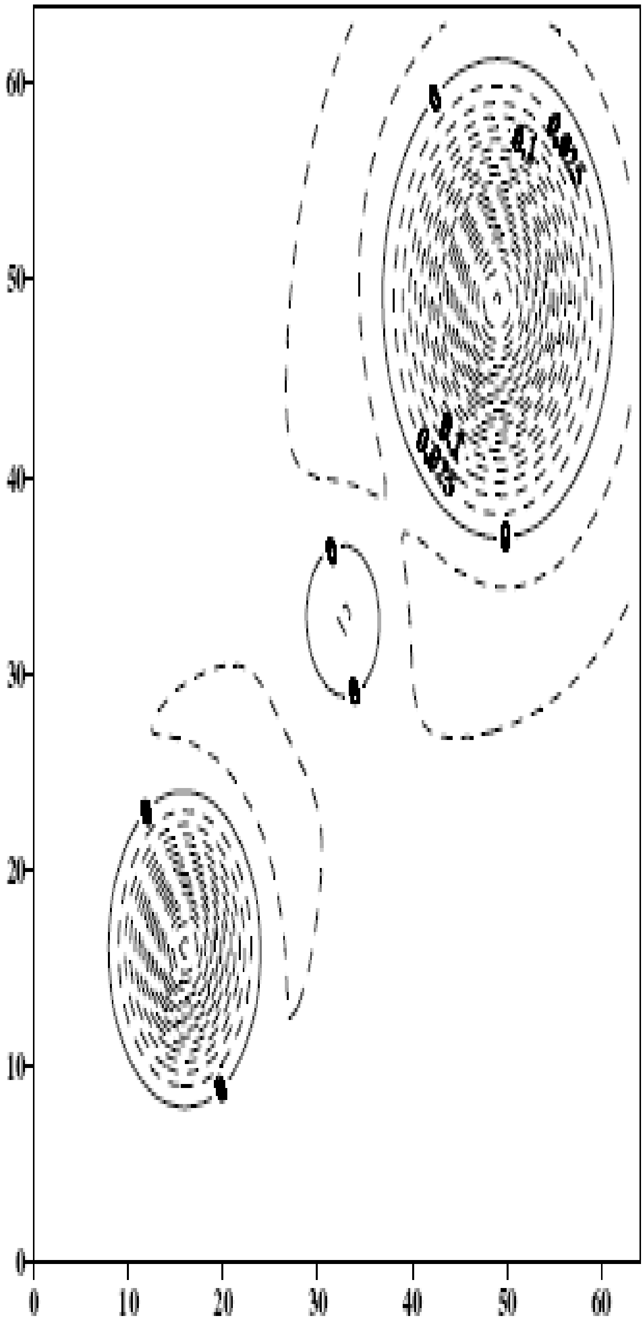

Fig. (12) The second derivative map for the complicated model by applying a simple 3x3 Laplacian filter.

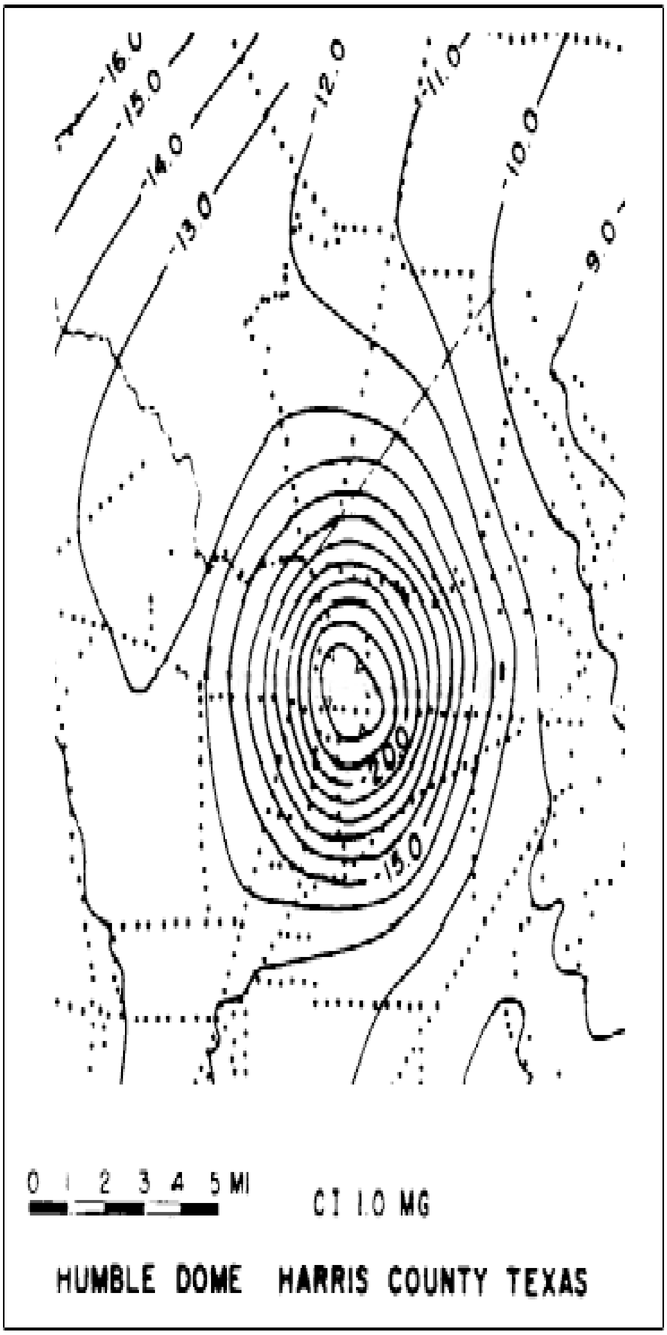

Fig. (13) Bouguer gravity map, Humble salt dome, Harris Country, Texas, USA, after (12).

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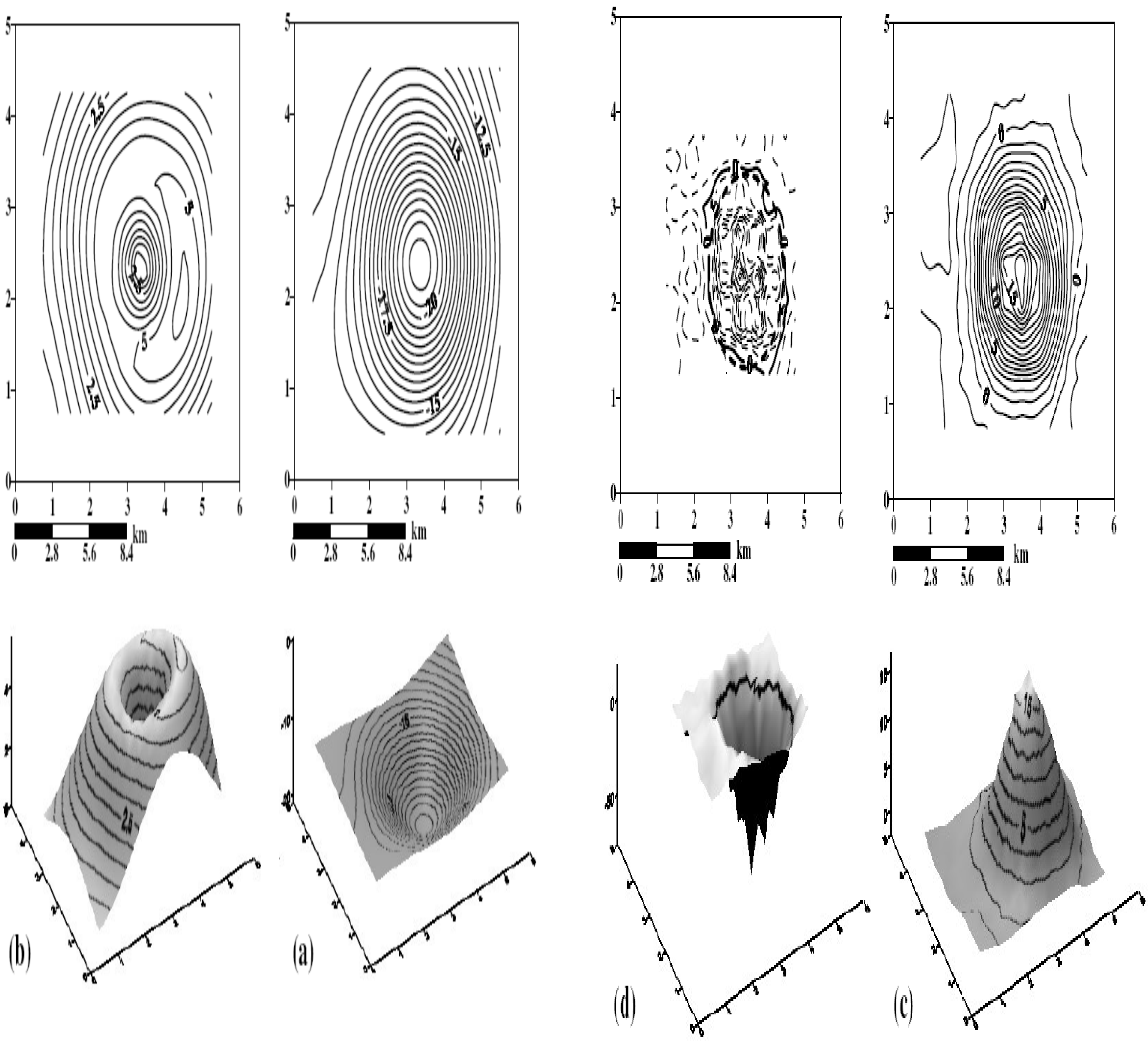

Fig (14) a- 2D and 3D digitized map for the Bouguer gravity map of Humble Salt Dome.
b- 2D and 3D presentation for the Gradient Operator.
c- 2D and 3D presentation for the Laplacian Operator.
d- 2D and 3D presentation for the Biharmonic Operator. The zero closed contour
define the depth to the center of the salt dome, its boundary and directions, the
dash line circle represent the mean estimation for depth.

تقدير العمق لاجسام كروية باستخدام المعاملات التفاضلية للانحدار، معاملات لابلاس والتوافقية المزدوجة لمجالاتها الجذبية.

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الخلاصة:

استخدمت المعاملات التفاضلية (الانحدار، لابلاس والتوافقية المزدوجة) لتحديد خصائص الشواذ لاجسام كروية ثلاثية الابعاد لها اعماق وانصاف اقطار وتباين كثافي مختلف ومن خلال مجالها الجذبي المحسوب نظريا. تقاطع المجال الجذبي مع المجالات التفاضلية المحسوبة يمكن ان تعين في حساب العمق الى مركز الاجسام الكروية بغض النظر عن الاختلاف في انصاف اقطارها واعماقها وتباينها الكثافي. معامل التوافقية المزدوجة اعطى نتائج ممتازة، حيث يعطي انغلاقين كنتوريين وبقيمة صفرية. قطر الانغلاق الداخلي ذي القيمة الصفرية يحدد العمق الى مركز الاجسام الكروية. هذه هي محاولة لاستخدام هذه الطريقة في حساب العمق، وللمعامل التوافقي المزدوج حساسية كبيرة لاظهار التراكيب الصغيرة والمخفية بتأثير الاجسام القريبة والكبيرة، مرشح لابلاس لحساب المشتقة الثانية يمكنه ايضا اظهار هذه التراكيب الصغيرة ولكن معامل التوافقية المزدوجة يمكنه حساب العمق. المستخدم لهذه الطريقة يجب ان يكون حذرا خلال حساب قراءاته لكون معامل التوافية المزدوجة حساس جدا لتغير هذه القراءات. صلاحية الطريقة اختبرت على قبة ملحية في الولايات المتحدة كمثال واعطت الطريقة نتيجة صائبة للعمق.