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| **Supra semi preopen sets in supra topological spaces**  **1N. A. Nadhim, 2Ahmed Hanoon Abud**  D:\مجلة\last\شعار المجلة.jpg1Department of Mathematics, Faculty of Education for pure science, University of AL-Anbar, AL-Anbar, Iraq.  2Department of Mathematics, College of Science, University of AL-Mustnsiriyah, Baghdad, Iraq | | |
| **ARTICLE INFO** | |  | **ABSTRACT** | |
| Received: 00 / 00 /2023  Accepted: 30 / 10 / 2023  Available online: 00 / 00 / 2023   |  | | --- | | DOI: 000000000000000000000 | | |  | The purpose of this article is to provide a new set which is supra semi preopen set via supra  topological space (, ). with the help of some examples and properties. Also, some of their properties have been investigate. With we illustrate the relationships between this concept and supra preopen set (respectively, supra open set).  Also, we define two spaces which are supra semi pre compact and supra semi pre Lindelöf spaces with the relationships between them. At the end of this work, we provide some examples, properties to support this work.  . | |
| **Keywords:**  *supra topological space, preopen set, supra preopen set, supra semi preopen set, supra pre compact space.*  Copyright©Authors, 2022, College of Sciences, University of Anbar. This is an open-access article under the CC BY 4.0 license ([http://creativecommons.org/licens es/by/4.0/](http://creativecommons.org/licens%20es/by/4.0/)). | |  |

**1-Introduction**

In 1983**,** Mashhour **[ 1 ]** introduced the concept of supra topological space, which is, for any set , and the collection of subset of that and X belong to, also arbitrary union of elements of is an element in. The pair is called supra topological space (briefly, su.top.sp), and the elements of are said to be supra open ( briefly, su.o) sets and its complements are supra closed ( briefly, su.c) sets. Also, he presented the relation between this concept and topological space ( for short top. sp.) which is (Every top.sp is su.top.sp).The auther in [2], the concept of supra interior was defined, the supra interior of a subset of supra space , which is su.int ={U: U , where U , and supra closure of ,that is su.cl ={: , where ,

After then in 2010, Sayed [3] provided a new concept which is supra preopen (respectively, supra pre closed) set, briefly, su. pr.o (respectively, su. pr. c) set, such that is su.pr.o if  su.int (su.cl()).

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The complement of a su. pr. o set is said to be su.pr.c, The collection of all su. pr. o (respectively, su.pr.c) sets of is denoted by su.pr.o () (respectively, su. pr.c ()). And this author defined, The union of all su.pr.o set contained in a subset of su.sp. is su.pr.interior of  and we denotel by su.intp Also, The intersection of all su.pr.c () sets containing is said to be su.pr closure of and we denoted by su.clp It is well known that top. sp and su. top. sp have been generalized and studied in many ways (see for example, [4], [5], [6]).

In 2020, Al-Shami and others [7], presented the concept of supra pre compact and supra pre Lindelöf spaces with many properties, examples and theorem with the relationships between these two spaces.

In this work, we provide a new concept which is supra semi\* preopen (briefly, su. pr.o) set in su.top.sp and the fact that is ″every su.o set is su. pr.o set ″, but the convers is not true, also the relation between su.pr.o set and su. pr.o set, see ( Remark 2.2, part(2)) in this research, Finally, we provide anew spaces namely su. pr. compact and su. pr. Lindelöf spaces (briefly, su. pr.com and su. pr. Lin) with some properties, examples for these spaces.

**2- supra.pre. open set**

At ,Veera Kumar [8], introduce the concept semi- pre-open set. By the same context via supra topology, we define the following: -

**Definition 2. 1: -** A subset of su. sp  is said to be su.pr. o, if there exists su. pr.oset., such that su.clp(

**Remarks 2. 2: -**

1-Every su. o set. is su.pr. o set. but the converse may be not true.

2- Every su. pr.o set is su.pr.o set , but the convers is not true.

**Example 2. 3: -** In su indiscrete sp., the set of all rational numbers is su.pr. o set. but not su.o set.

**Example 2. 4**: - Let .

Then,

the

and su.pr. o () ,

we note that {1,2} is su.pr. o set but not set.

**Proposition 2. 5: -** The union of any family of su.pr.o () is su.pr. o ()

Proof: - Let be a family of su su.pr. o set of , so there exists pr.o subset such that  clp , for each .

Also, from (theorem2 [3]) ( ) su.cl()

Then, is su.pr. o set.

**Corollary 2. 6: -** The intersection of any family of su.pr.c () is su.pr.c ().

Proof:- Let be su.pr.o sets, by (theorem2 [3]), we get be su.pr.o set, but =,

so = is su.pr.cl ().

**Remark 2. 7: -** The intersection of two su.pr.o sets is may be not su.pr.o set.

**Example 2. 8:** I n (Example2.4) the intersection of two su.pr.o sets is not su.pr.o set.

Proof: - Suppose is su. pr.o set andsu.clp(), then it is su.pr.o set .

**Proposition 2. 9**: - A subset of su.sp is su.pr.c set if and only if su.pr.cl(.

Proof: - Let be su.pr.c set, the smallest intersection of all su.pr.c sets which contain is equal to . Therefore, = su. s\* pr.cl ().

Conversely, since su.pr.cl always su.pr.c set, by corollary (2.6), Then, is also su.pr.c set.

**Corollary 2. 10: -** For a subset of su.sp , we have su.pr.cl ( su.pr.cl()) = su.pr.cl()

Proof: - Since by corollary (2.6), is su.pr.c set, then by (proposition 2.9), we get the result.

**Definition 2. 11: -** A point , is said to be su.pr. interior point to a subset of su.sp , if there is su.pr.o set with .

**Definition 2. 12: -** A point , is said to be su.pr-adherent point to a subset of su.sp , ifwhere is su.pr.o set containing .

**Proposition 2. 13: -**For a subset of su.spwe have su.pr.intsu.pr.cl(

Proof:- Suppose  su.pr.cl( ,so su.pr.cl(, then for each su.pr.o set containing , we get  that is is not su.pr. interior point to , that is su.pr.int(), thensu.pr.int(su.pr.cl()

Conversely, Let su.pr.int, then there is su.pr.o set containing with , then **,** so is su.pr-adherent point to , that is su.pr.cl( .Hence, su.pr.cl(, thensu.pr.cl( su.pr.int(). Hence, su.pr.intsu.pr.cl (

**Proposition 2. 14: -** A subset of is su.pr.o set if and only if su.pr. int.

Proof: - Let su.pr. int, but the union of su.pr.o set is also su.pr.o set, then su.pr. intis su.pr.o set, which equal to , then  is su.pr.o set.

Conversely, suppose is su.pr.o set, so it is su.pr-neighborhood of each its point, so su.pr. int.

**3- su.pr - Lindelöf space**

In this section, we introduce new concepts namely su.pr-compact space (briefly, su.pr-com. sp) and su.pr-Lindelöf space (briefly, su.pr-Lind. sp).

**Definition 3. 1: -** A su.sp is said to be su.pr-com.sp , if for each su.pr.o cover to has a finite subcover.

**Definition 3. 2: -** A su.spis said to be su.pr-Lind. sp, if for each cover to has a countable subcover.

**Remark 3. 3: -** Every su.pr-com sp is su.pr-Lind. sp.

**Proposition 3. 4: -** Every su.pr.c subset of su.pr-Lind. sp is su.pr-Lind.

**Proof**: - Let be su.pr.c subset of su.pr-Lind. sp and be su.pr.o cover to , that is , so ) ∪

, where  is su.pr. o set, but is su.pr-Lind., then Hence, , then is su.pr-Lind.

**Proposition 3.5: -**The union of two su.pr-Lind. subsets of su.sp is su.pr-Lind

**Proof**: - Let and be two su.pr-Lind. subsets and }be su.pr. o cover to , that is, , then C is su.s\*po cover to H and K, then H and K, since H and K are su.s\*p- Lind. set. So, HK .

Hence, HK is su. s\*p- Lind. set.

**Proposition 3. 6: -**A su. sp is su. s\* p- comp. if and only if every collection of su.s\*pc set of with FIP has non empty intersection.

**Proof**:- Let be su.s\*p- comp. and{Gα :α∈Ω }be su.s\*pc subsets of with assume that  , then, then, but for each, we have is su.s\*po set, so { be a cover of su.s\*po sets of , which is su.s\*p- comp., then , so . Hence, , which is a contradiction with FIP, therefore .

**Proposition 3. 7: -** Let (, ) be su. sp, then every subspace of is su. s\*p- Lind., if every su.s\*po of is su.s\*pr- Lind.

**Proof**: - Let H be any set in and { :α∈Ω }be su. s\*po cover to H. So, H, by proposition 2.5, is su. s\*po, hence it is su. s\*p- Lind. , then H⊆. Therefore, H is su. s\*p- Lind..

**Conclusion**

In this work, we presented a type of a set, which isSupra semi preopen sets in supra topological spaces,

with some characteristics, examples, and theories associated with that sets. Whoever reads this work should consider other groups in the topological space such as Supra ω- preopen set in supra topological spaces or Supra θ- preopen set in supra topological spaceswith proceed with the same research method to reach what is desired.

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**المجاميع المفتوحة من النمط شبه الفوقية في الفضاءات الطبولوجية الفوقية**

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**الخلاصة:**

الغرض من هذا البحث هو تقديم مجموعة جديدة هي **ا**لمجاميع المفتوحة من النمط شبه الفوقية في الفضاءات الطوبولوجية الفوقيةبمساعدة بعض الأمثلة والخصائص. كما تم التحقيق في بعض تلك الخصائص. مع توضيح العلاقات بين هذا المفهوم والمجموعة المفتوحة أعلاه (على التوالي، المجموعة المفتوحة من النمط شبة الفوقية).

أيضًا، قمنا بتعريف فضاءين هما الفضاء شبه المرصوص الاعلى وفضاء شبه اللايندلوف الاعلى مع توضيح العلاقات بينهما. وفي نهاية هذا العمل، قدمنا بعض الأمثلة والخصائص لدعم هذا العمل.

**الكلمات المفتاحية**: - الفضاء الطوبولوجي الفوقي. المجموعة قبل المفتوحة. المجموعة قبل المفتوحة الفوقية. المجموعة قبل المفتوحة شبه الفوقية. الفضاء المرصوص قبل الفوقية.