# TOTAL CROSS SECTION FOR PHOTON- PHOTON INTERACTIONS 

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#### Abstract

Regge model has been used to calculate photon - photon total cross section. The couplings of the exchanged soft pomeron, hard pomeron and Reggeons with photons have been calculated. It has been assumed that soft and hard pomerons couple to photons in away similar to that of the pseudo scalar mesons. A comparison with data on photon-photon total cross section shows a good agreement.


## INTRODUCTION

It is well known that the hadronic total cross sections increase with energy above 10 GeV . At lower energies these cross sections decrease with energy. The behavior in this region is different from one process to the other. Thus at high energies it is possible to compare models, which describe the proton - proton, proton - antiproton, photon - proton and photon -photon total cross sections.

Models that used to describe the photon - photon total cross section can be classified into two main groups [1]. The first group involves models, which depend on Regge theory. Photons and hadrons exchange Reggeons and pomerons. Reggeons describe the initial decrease of the cross section with energy while pomerons describe the subsequent rise with increasing energy. This group also involves the factorization models where a simple constant allows us to move from one process to the other and the scaling models where the various hadronic processes are related through the dimension counting rule.

[^0]The second group involves the QCD models. These models ascribe the rise of the total cross section to parton - parton scattering. Some of these models used as well higher order QCD effects, soft gluon summation and the transverse momentum distribution of partons.

According to Regge model the total cross section is given by [1]
$\sigma=Y_{a b} s^{-R}+X_{a b} s^{\varepsilon}$
Where $R$ and $\varepsilon$ are the intercepts at zero of the leading Regge trajectory and pomeron respectively. Their values are usually taken as $R=0.476$ and $\varepsilon=0.08$. These powers were assumed to have the same values for all hadronic processes. The coefficients in (1) obey the following factorization khalid hamad2002@yahoo.com

$$
\begin{equation*}
X_{a a} X_{b b}=X_{a b}^{2} \quad \text { and } \quad Y_{a a} Y_{b b}=Y_{a b}^{2} \tag{2}
\end{equation*}
$$

For $\sigma_{p p}$ and $\sigma_{p p}$ these coefficients are usually extracted by data fitting. The corresponding coefficients for $\sigma_{r \gamma}$ then can be obtained through (2). Experimental data on $\sigma_{\gamma \gamma}$ for $w \leq 10 \mathrm{GeV}$ [2] where the high energy rise could not have been seen and for
$w$ around 50 GeV can be represented by Regge form in (1) (see also fig.2). At higher energies [3,4] the form in (1) does not follow the trend of the data. Donnachie and Landshoff fit[5] for L3 data[6] and OPAL[7] data indicated that a third term should be added to (1). This term was attributed to hard pomeron exchange. The general form of Regge equation should be written as $\sigma=Y_{a b} s^{-R}+X_{1 a b} s^{\varepsilon_{1}}+X_{0 a b} s^{\varepsilon_{0}}$

The powers were taken as $R=0.476$, $\varepsilon_{1}=0.0667$ and $\varepsilon_{0}=0.452$ for Reggeons, soft and hard pomerons respectively. Hard pomeron has also been observed in the fit of the proton structure function [8] and the charm structure function [9]. An intercept around $1+\varepsilon_{0}=1.4$ was needed to fit the data. The coefficients in (3) will be calculated for real photons interactions. This can be done simply by calculating the couplings of the photons with the exchanged Reggeons and pomerons. The effect of the photon mass was discussed in ref [5] although more work should be done.

## 2-Photon-Photon Total Cross Section

The amplitude of elastic photon-photon scattering is given in fig.1. The dotted line represents the Reggeons and pomerons which couple to photons through the quark loop. In general the Regge amplitude [10] for $a b \rightarrow c d$ process is given

$$
\begin{equation*}
A(s, t)=g_{a c}(t) g_{b d}(t) \zeta(t) s^{\alpha(t)} \tag{4}
\end{equation*}
$$

Here $g_{a c}(t)$ and $g_{b d}(t)$ represent the coupling of the trajectory to aand $c$ at the upper vertex and to $b$ and $d$ at the lower vertex. The factorization property of the coupling is included. The form of $\zeta(t)$ is taken as:

$$
\begin{equation*}
\zeta(t)=\frac{\exp (-i \pi \alpha(t))+L}{\sin \pi \alpha(t)} \frac{1}{\Gamma(\alpha(t)} \tag{5}
\end{equation*}
$$

Then the total cross- section is given by

$$
\begin{equation*}
\sigma=g_{a c} g_{b d} s^{\alpha(0)-1} c_{1} \tag{6}
\end{equation*}
$$

Where g is the coupling at $\mathrm{t}=0$ and $\alpha(0)$ is the intercept of the trajectory at $\mathrm{t}=0$. The constant $c_{1}$ is nearly one for soft pomeron, $\frac{2}{\sqrt{\pi}}$ for hard pomeron and $\frac{1}{\sqrt{\pi}}$ for Reggeons.

## 2-1 Soft Pomeron - Photon Coupling

We notice that the pomeron-two photons vertex in fig .1 is similar to the $\pi^{0}$-two photons vertex discussed by 't Hooft and Veltmann [11]. Therefore their results can be used directly in our calculation. We just need to replace the pion-quark coupling by pomeron -quark coupling. The trace calculation of the quark loop gives

$$
\begin{equation*}
G_{p r \gamma}=\sum_{q} 32 \pi \alpha n \text { g } e_{q}^{2} m_{q} G\left(m_{1}, m_{2}, m_{3}\right) \tag{7}
\end{equation*}
$$

Where $n=3$ is the number of colour and $m_{q}$ and $e_{q}$ are the mass and the charge of the quark respectively. g is a dimensionless coupling constant of the pomeron to the quarks. The summation is over all types of the quarks. The factor $G\left(m_{1}, m_{2}, m_{3}\right)$ is the integration over the quark loop given in terms of the three external masses connected to the triangle. This integration was performed by the authors [11] in terms of three scalar point techniques. For real external masses the integral is given by a symmetric function involving Spence function. If one of the external masses is zero the integral reduces to an elementary function. Then for real photons ( $m_{1}=m_{2}=0$ ) and scattering with transfer momentum $-t<0$ we have
$G(0,0, t)=\frac{1}{(4 \pi)^{2} t} \frac{1}{2} \ln ^{2}\left(\frac{\gamma+1}{\gamma-1}\right)$

With $\quad \gamma=\sqrt{\left(1+\frac{4 m_{q}^{2}}{t}\right)}$
Then

$$
\begin{equation*}
\ln ^{2}\left(\frac{1+\frac{1}{\gamma}}{1-\frac{1}{\gamma}}\right)=\ln ^{2}\left(\frac{1+y}{1-y}\right) \tag{10}
\end{equation*}
$$

We notice that as $t \rightarrow 0$ then $y \rightarrow 0$ therefore, we can expand the function in (10) in terms of Maclaurin expansion: $\ln ^{2}\left(\frac{1+y}{1-y}\right)=4\left(y^{2}+2 y^{3}+y^{4}+\ldots \ldots\right)$

Then equation (8) reduces to

$$
\begin{align*}
& G(0,0, t)=\frac{1}{8 \pi^{2}\left(t+4 m_{q}^{2}\right)}\left(1+\frac{\sqrt{t}}{\sqrt{\left(t+4 m_{q}^{2}\right)}}\right. \\
& +\ldots \ldots \ldots . .) \tag{12}
\end{align*}
$$

We notice that (12) gives the transfer momentum dependence of the form factor. Since we are interested in the region where $t=0$ then equation (12) gives

$$
\begin{equation*}
G(0,0,0)=\frac{1}{32 \pi^{2} m_{q}^{2}} \tag{13}
\end{equation*}
$$

Substituting (13) in (7) we get the following equation for the soft pomeron-photons coupling

$$
\begin{equation*}
G_{P \gamma \gamma}=\frac{3 \alpha}{\pi} \sum_{q} e_{q}^{2} \frac{g}{m_{q}} \tag{14}
\end{equation*}
$$

This is similar to the $\pi^{0} \rightarrow \gamma \gamma$ coupling constant in the mass less limit but the summation is now carried over all types of the quarks. Suppose that $\beta$ is a dimensional quantity with $\beta=\frac{g}{m_{q}}$. If $\beta$ were the average coupling of the pomeron to the quarks then

$$
\sum_{q} e_{q}^{2} \frac{g}{m_{q}} \text { would equal to } \mathrm{c} \beta \text { with } \mathrm{c} \text { is around 1.2. }
$$

Assuming that $\beta$ is the coupling of the pomeron to up or down quark then the quantity $\sum_{q} e_{q}^{2} \frac{g}{m_{q}}$ can be
expanded in terms of $\beta$ so $\sum_{q} e_{q}^{2} \frac{g}{m_{q}}=c \beta$ with c is the sum of the coefficients of $\beta$ in the expansion. Then

$$
\begin{equation*}
G_{P \gamma \gamma}=\frac{3 \alpha}{\pi} c \beta \tag{15}
\end{equation*}
$$

With c is around one. The value of $\beta$ can be calculated from the proton- proton total cross sections in the pomeron region. Using (6) then $\sigma_{p p}^{P}=9 \beta^{2} s^{\varepsilon_{1}}$ The contribution from the soft pomeron can be given as

$$
\begin{equation*}
\sigma_{\gamma \gamma}^{P}=\left(\frac{c \alpha}{\pi}\right)^{2} \sigma_{p p}^{P} \tag{16}
\end{equation*}
$$

For $c=1.25$ we get

$$
\begin{equation*}
\sigma_{\gamma}^{P}=204.5 s^{\varepsilon_{1}} \quad n b \tag{17}
\end{equation*}
$$

## 2-2 Hard Pomeron - Photons Coupling

It has been found [5] that hard pomeron is a flavour blind. It couples equally to all types of the quarks then (14) can be written in the form

$$
\begin{equation*}
G_{H P \gamma \gamma}=\frac{3 \alpha}{\pi} n \beta_{H} \tag{18}
\end{equation*}
$$

Where n is the number of the quarks involved in the process and $\beta_{H}$ is the hard pomeron-quark coupling. The value of $\beta_{H}$ can be calculated from the protonproton total cross section in the hard pomeron region as well. Therefore, the contribution from the hard pomeron to the cross section can be written as:

$$
\begin{equation*}
\sigma_{\gamma \gamma}^{H P}=\left(\frac{n \alpha}{\pi}\right)^{2} \sigma_{p p}^{H P} \tag{19}
\end{equation*}
$$

For $n=5$ (i.e. up to the b-quark) and with $\sigma_{p p}^{H P}=0.0139 s^{\varepsilon_{0}} m b \quad[5]$ we get $\sigma_{\gamma \gamma}^{H P}=1.877 s^{\varepsilon_{0}} n b$

For $\mathrm{n}=6$ slightly higher values for $\sigma_{p p}^{H P}$ can be obtained.

## 2-3 Reggeons-Photons Coupling

Reggeons that could couple with two photons are particles with even charge conjugation. We consider here the leading Reggeons $f, f^{\prime}$ and $A$. In the decay of $f$ meson into two photons we should consider two independent amplitudes corresponding to $\lambda=0$ and $\lambda=2$ spin states. The coupling constant is given by $[12] g_{f r y}^{2}=(1+2 \lambda) \frac{64 \pi \Gamma_{f r y}}{m_{f}^{3}}$

Although the amplitude $\lambda=2$ dominates the experimental data $[12]$ we shall consider here the average of the both amplitudes then for $\Gamma_{f y \gamma}=3 \mathrm{KeV}$ we get $g_{f r \gamma}=18.57 \times 10^{-3} m b^{\frac{1}{2}}$. The other coupling constants are given by the relation $g_{f \gamma \gamma}^{2}: g_{A \gamma \gamma}^{2}: g_{f^{\prime} \gamma \gamma}^{2}=25: 9: 2$

Then the contribution from Reggeons to the photon photon total cross section is given as

$$
\begin{equation*}
\sigma_{\gamma \gamma}^{R}=1.44 \sigma_{\gamma \gamma}^{f}=280.5 s^{-R} n b \tag{23}
\end{equation*}
$$

Finally the contribution from the quark box [13] diagram should be added to the photon-photon total cross section.

## 3- DISCUSSION

The predictions from our model (curve 2) compared with data $[3,4]$ and some theoretical models are given in fig.2. The contribution from the soft pomeron alone
(curve 1) is appeared in the figure. We notice that the hard pomeron becomes effective above $w \approx 50 \mathrm{GeV}$. The model of Schuler and Sjostrand is shown. According to this model the photon has VMD component, anomalous and QCD contributions. The model of Engel and Rafnt is also shown. More details on these models are given in ref. [1]. The predictions from our model agree nicely with the data. Furthermore fits [5] for total cross sections are given as follow: $\sigma_{p p}=46.55 s^{-R}+24.22 s^{\varepsilon_{1}}+0.0139 s^{\varepsilon_{0}}$

$$
\begin{equation*}
\sigma_{p p}=0.113 s^{-R}+0.0737 s^{\varepsilon_{1}}+0.000169 s^{\varepsilon_{0}} \tag{24}
\end{equation*}
$$

Using the factorization in (2) we get

$$
\begin{align*}
\sigma_{\gamma \gamma}= & 274 \times 10^{-6} s^{-R}+224.3 \times 10^{-6} s^{\varepsilon_{1}} \\
& +2.055 \times 10^{-4} s^{\varepsilon_{0}} \tag{26}
\end{align*}
$$

The results of our calculations are

$$
\begin{align*}
\sigma_{r \gamma}= & 280.5 \times 10^{-6} s^{-R}+204.5 \times 10^{-6} s^{\varepsilon_{1}} \\
& +1.877 \times 10^{-4} s^{\varepsilon_{0}} \tag{27}
\end{align*}
$$

All the above coefficients are given in $m b$. The quark box diagram should be added to (26) and (27). Compare the figures in these two equations we find that our calculated figures in (27) are nearly equal to the corresponding figures obtained from data fit in (26). Since the factorization is a characteristic of Regge poles then this result is clearly consistent with our calculations which treated the soft and hard pomerons as simple poles.

## REFERENCES

[1] A. Corestti, R.M. Godbole and Pancheri, hepph/9707360; R. M. Godbole, A. Grua and Pancheri, hep-th/9908220.
[2] PLUTO Collaboration: Ch. Berger et al., Phys. Lett. B149 (1984) 421; TPC/2 $\gamma$ Collaboration: H. Aihara et al., Phys. Rev. D41 (1990)2667; S. E. Baru et al. Z- Phys. C53 (1992)21.
[3] L3 Collaboration: M. Acciarri et al., CERN PPE-97-48.
[4] F. Wackerle, hep-ex /9710005.
[5] A. Donnachie and P. Landshoff hep-ph/0402081. and Phys. Lett. B595 (2004) 393.
[6] L3 Collaboration: Phys. Lett. B519 (2001) 33.
[7] G. Abbendi et al., European journal C14 (2000) 199.
[8] A. Donnachie and P. Landshoff, Phys. Lett. B437 (1998) 408 and B518 (2001) 63.
[9] L3 Collaboration: M Acciari et al., Phys. Lett. B514 (2001) 19.
[10] P. Collins and A. Martins, Hadrons interactions, (Adam Hilger, Bristol, 1984).
[11] G. 't Hooft and M. Veltemann, Nucl. Phys. B153 (1979) 39 ; K. M. Hamad Journal of Al-Anbar Un. V.1(2007) 39.
[12] Paul Singer, Phys. Lett. B124 (1983) 531.
[13] M. Greco and Y. Srivastave, Nuovo Cim. 43A (1978) 88.


Fig. 1 schematic represntation for elastic photon-photon scattering. The triangle represents the pomeron - photon vertex.


Fig. 2 the $\sigma_{\gamma \gamma}$ total cross sections against $w$. Our model is given by the curve (2).The contribution from the soft pomeron alone is given by the curve (1). The data and the theoretical models are given in ref. (4).


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$$
\begin{aligned}
& \text { تـم اسـتخدام نمـوذج رجـي لحسـاب المقطـع العرضـي الكلـي لتصــادم الفوتون مـع الفوتـون. حيث تـم حسـاب ازدواج البـومرون النـاعم } \\
& \text { والبومرون الصلب و الرجونات مع الفوتونات. لقد تم افتراض ان البومرون الناعم والبومرون الصلب تزدوج مـع الفوتونات بطريق مشابة لازدواج } \\
& \text { الفوتونات مع الميزونات ذات العديدبة المصطنعة. لقد اضهرت المقارنة نوافقا جيدا بين نتائج العملية والنضرية. }
\end{aligned}
$$


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