# PHOTON-PROTON TOTAL CROSS SECTION AND FACTORIZATION PROCESS 

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#### Abstract

A fit for photon-proton total cross section ( ${ }^{\sigma_{p}}$ ) data in terms of Regge model has been obtained. As usual the fit can be obtained by calculating the coupling constants of the exchanged Reggeons and pomerons in the process. The model has three Regge components, the low energy region is dominated by $f$ meson and the high energy region is dominated by the soft and hard pomerons. As Regge pole factorizes the hadronic total cross sections are related by factorization. A fit for the photon -photon total cross section ( $\sigma_{\gamma \gamma}$ ) can also be obtained by the factorization.


## Introduction

It is quite clear from data on $\sigma_{\bar{p} p}, \sigma_{p p}[1]$, $\sigma_{\gamma p}[2-4]$ and $\sigma_{\gamma \gamma}[5-7]$ that hadronic total cross sections increase with increasing the c.m. energy ${ }^{(w)}$ at high energies. They initially decrease with energy then subsequently start to increase as the energy increases. Thus one can compare models and their predictions for $\sigma_{\bar{p} p}$ and $\sigma_{p p}$ to those models for $\sigma_{\gamma p}$ and $\sigma_{\gamma \gamma}$. According to the QCD models ${ }^{[8-9]}$ the rise in the total cross sections can be attributed to partons interactions in lowest order and higher order corrections. All these models are based on the eikonal formalism where eikonal is function of the impact parameter. It receives contributions from partons interactions. The way of driving the eikonal is different from model to another. These models are usually formulated for proton - proton scattering. Extending these models to photon- proton and photonphoton processes is also different from model to

[^0]another $[8-10]$. Regge type models assume that the initial decrease of the total cross sections and the subsequent increase are due to exchange of Reggeons and pomerons. It has become tradional to believe that soft hadronic processes are dominated by the exchange of soft pomeron ${ }^{[11]}$ at high energy with Regge intercept close to ${ }^{1+\varepsilon_{1}}=1.08$. However, analysis [12] of the hard scattering processes has revealed the presence of hard pomeron as well, with intercept close to ${ }^{1+\varepsilon_{0}}=1.4$. Donnachie and Landshoff $[12]_{\text {have }}$ reanalyzed the data on $\sigma_{\bar{p} p}, \sigma_{p p}, \sigma^{\gamma p}$ and the proton structure function using the minimum $\chi^{2}$ method. They concluded that involving of a hard pomeron would reduce the soft pomeron intercept. According to their fits the hadronic total cross sections can be given by the following universal Regge form:
\[

$$
\begin{equation*}
\sigma=Y s^{-R}+X_{1} s^{\varepsilon_{1}}+X_{0} s^{\varepsilon_{0}} \tag{1}
\end{equation*}
$$

\]

Where R, $\varepsilon_{1}$ and $\varepsilon_{0}$ are the intercepts of the leading Regge trajectory and of the soft and hard pomerons, respectively $\mathrm{R}=0.476$,
$\varepsilon_{1}=0.0667$ and $\varepsilon_{0}=0.452$. The coefficients in the above equation are extracted from the fits. As these coefficients are related to the couplings of the exchanged trajectories with the interacting particles, our strategy is to calculate these couplings. Previously ${ }^{[13]}$ we have derived an expression for the soft pomeron-photons coupling. As the other couplings were not available our fit for $\sigma_{\gamma p}$ was limited to the soft pomeron region. Since $[14]_{\text {we have extended the }}$ calculation to include the hard pomeron -photons and the Reggeon- photons couplings a complete fit for $\sigma_{r \gamma}$ was obtained. In the present work we are intending to get a complete fit for ${ }^{\gamma p}$. As the hard pomeron- photons coupling is now available we only need to get the Reggons- proton couplings. Furthermore, we shall investigate the factorization process. A fit for $\sigma_{\gamma \gamma}$ in terms of this process will be discussed.

## Reggeon - proton couplings

According to the fits given by ref. [12] $\sigma_{p p}$ and $\sigma_{\bar{p} p}$ have the following forms:

$$
\begin{align*}
\sigma_{p p} & =46.55 s^{-R}+24.22 s^{\varepsilon_{1}} \\
& +0.0139 s^{\varepsilon_{0}}  \tag{2}\\
\sigma_{\bar{p} p} & =95.81 s^{-R}+22.4 s^{\varepsilon_{1}} \\
& +0.0139 s^{\varepsilon_{0}} \tag{3}
\end{align*}
$$

The above coefficients are given in mb. We notice that the coefficient in the low energy region in eq. (3) is higher than that in eq.(2). This can be explained by assuming that there is a cancellation
between the imaginary parts of two exchanged Reggeons in the pp elastic amplitude $[15]$.

Since $\sigma_{p p}=\sigma_{n p}$ the exchanged Reggeon must be made of $u \bar{u}$ and $d \bar{d}$ in isospin zero combination i.e. $\frac{1}{\sqrt{2}}(u \bar{u}+d \bar{d})$. This couples equally to neutrons and protons. Reggeons that have such kind of combination are $f$ and $\omega$. Therefore , $\sigma_{\bar{p} p}$ exchanges $f$ and $\omega$ at low energies and pomerons at high energies. Then we may write

$$
\begin{equation*}
\sigma_{p \bar{p}}=f+\omega+\text { pomeron }_{\mathrm{s}} \tag{4}
\end{equation*}
$$

As $\omega$ has spin 1 and odd charge conjugation it contributes oppositely to $\sigma_{p p}$ then

$$
\begin{equation*}
\sigma_{p p}=f-\omega+\text { pomeron }_{\mathrm{s}} \tag{5}
\end{equation*}
$$

Since the contribution from $f$ and $\omega$ to $\sigma_{p p}$ is not zero the cancellation is not exact. This means that the coupling constants of $f$ and $\omega$ are not equal. Moreover, the contribution from $\rho$ and $A$ mesons can be neglected because their isospin is one. We notice that

$$
\begin{align*}
& f-\omega=46.55 \mathrm{mb}  \tag{6}\\
& f+\omega=95.8 \quad \mathrm{mb} \tag{7}
\end{align*}
$$

Adding and subtracting the above equations we get the contributions from $f$ and $\omega_{\text {as }}$

$$
\begin{align*}
& \sigma_{p p}^{f}=71.175 \mathrm{~s}^{-R} \quad \mathrm{mb}  \tag{8}\\
& \sigma_{p p}^{\omega}=24.63 \mathrm{~s}^{-R} \quad \mathrm{mb} \tag{9}
\end{align*}
$$

As Regge residue factorizes, the above coefficients are related to the square of the coupling constants.

## Photon-proton total cross section

Consider the process $\gamma p \rightarrow x_{\text {where a photon }}$ interacts with a proton to produce hadrons. The cross section is given by squaring the amplitude, summing over the final state x and dividing by the invariant phase (2s) as given in fig. (1a). The optical theorem relates the total cross section to the imaginary part of the forward elastic amplitude at zero transfer momentum ( $\mathrm{t}=0$ ) as in fig. (1b). Appealing to Regge theory the discontinuity in the elastic amplitude can be represented by Reggeon / pomeron exchange at high energy as given in fig. (2). The dotted line in the figure repesents the exchanged Reggeon or pomeron. The vertex of their coupling with photons is represented by the triangle. According to Regge model the amplitude in fig.(2) is given as

$$
\begin{equation*}
A(s, t)=g_{\gamma \gamma}(t) g_{p p}(t) \zeta(t) s^{\alpha(t)} \tag{10}
\end{equation*}
$$

Where
$\zeta(t)=\frac{\exp (-i \pi \alpha(t))+L}{\sin \pi \alpha(t)} \frac{1}{\Gamma(\alpha(t))}$
$g_{\gamma \gamma}(t)$ is the coupling of the exchanged Reggeon or pomeron with photons at the upper vertex, $g_{p p}(t)$ is the coupling with the proton at the lower vertex and $\alpha(t)$ is the trajectory. Recall that the optical theorem involves only the imaginary part of the forward elastic amplitude, then
$\sigma_{\gamma p}=g_{\gamma \gamma}(0) g_{p p}(0) s^{\alpha(0)-1} c$
Where
$c=(\operatorname{Im} \zeta(t))_{t=0}$
The above constant can be easily calculated from the Gamma function. Using $0.5,1.0$ and 1.5 as approximate intercepts for Reggeon, soft pomeron and hard pomeron, the values of c are $\frac{1}{\sqrt{\pi}}$, land $\frac{2}{\sqrt{\pi}}$
respectively for Reggeon, soft pomeron and hard pomeron. At the upper vertex we have Reggeonphotons coupling. Reggeons that may couple to two photons are particles with even charge conjugation. Only leading Reggeons are considered here i.e. $f, f^{\prime}$ and $A$. But as we said proton couples mainly to $f$ and $\omega$. Since $\omega_{\text {has odd charge conjugation it }}$ cannot couple to two photons. Thus, in fig. (2) Only $f$ meson can contribute. Applying eq. (12) for pp scattering with $\sigma_{p p}=71.715 s^{-R}$ we get
$g_{f p p}=11.23 m b^{\frac{1}{2}}$
As $[14] g_{f r \gamma}=18.57 \times 10^{-3} m b^{\frac{1}{2}}$ then eq. (12) gives $\sigma_{\gamma p}^{f}=117.7 \times 10^{-3} s^{-R} \quad m b$

At high energy the process in fig.(2) is dominated by soft and hard pomerons exchange. The coupling of soft pomeron with photons is given as $[13]$ $g_{P \gamma \gamma}=\frac{3 \alpha c \beta}{\pi}$

With $\mathrm{c}=1.25$ and $\beta$ is the coupling of the soft pomeron with quark. At the lower vertex the soft pomeron couples to the proton. Since there are three valance quark in the proton this coupling is $3 \beta$. The value of $\beta$ can be fixed by $\sigma_{p p}$ in the soft pomeron region Then from eq. (12) we get:

$$
\sigma_{p p}^{P}=\frac{\alpha c}{\pi} 9 \beta^{2} s^{\varepsilon_{1}}
$$

$$
\begin{equation*}
=0.07037 s^{\varepsilon_{1}} m b \tag{17}
\end{equation*}
$$

The coupling of the hard pomeron with photons is

$$
\begin{equation*}
g_{H p \gamma \gamma}=\frac{3 n \alpha \beta_{H}}{\pi} \tag{18}
\end{equation*}
$$

where n is the number of the quarks at the upper vertex and $\beta_{H}$ is the coupling of the hard pomeron with quark. As before the pomeron is free to couple to all types of the quarks then (n) should equal 6. But $[14]$ with $\mathrm{n}=5$ a better agreement with Donnachie and Landshoff was found [12]. At the lower vertex the coupling of the hard pomeron with the proton is $3 \beta_{H}$. The contribution from the hard pomeron is

$$
\begin{equation*}
\sigma_{p}^{H P}=161.6 \times 10^{-6} s^{\varepsilon_{0}} \mathrm{mb} \tag{19}
\end{equation*}
$$

The general form of the photon- proton total cross section can be written as

$$
\begin{align*}
& \sigma_{x_{p}}=117.7 \times 10^{-3} s^{-R}+70.37 \times 10^{-3} s^{\varepsilon_{1}} \\
& +161.6 \times 10^{-6} s^{\varepsilon_{0}} \tag{20}
\end{align*}
$$

While that of Donnachie and Landshoff [12] is given by:

$$
\begin{align*}
& \sigma_{p p}=113 \times 10^{-3} s^{-R}+73.7 \times 10^{-3} s^{\varepsilon_{1}} \\
& +169 \times 10^{-6} s^{\varepsilon_{0}} \tag{21}
\end{align*}
$$

We notice that our fit is in good agreement with that of Donnachie and Landshoff.

## Factorization

The factorization hypothesis implies that Regge residue of a pole can be factorized into two factors. Each factor is a vertex representing the coupling of the exchanged Reggeon with the particles at that vertex. Accordingly for each exchanged Reggeon the coupling constants in $\sigma_{p p}, \sigma_{p \text { and }} \sigma_{\gamma \gamma}$ are related such that

$$
\begin{equation*}
\sigma_{y \gamma}=\frac{\sigma_{p}^{2}}{\sigma_{p p}} \tag{22}
\end{equation*}
$$

It is clear that the exchanged Reggeon should contribute to all of the three cross sections. As we have seen that only the $f$ meson and the pomerons are contributing to all of the three cross sections. The other Reggeons like $f^{\prime}$ and $A$ are contributing to $\sigma_{\gamma}$ and $\omega$ is contributing to ${ }^{\sigma}{ }_{p p}$. Applying eq.(22) for each exchanged trajectory we get the following values

$$
\begin{gather*}
\sigma_{\gamma}^{f}=193.1 \times 10^{-6} s^{-R}, \\
\sigma_{\gamma \gamma}^{P}=204.5 \times 10^{-6} s^{\varepsilon_{1}} \quad \text { And } \quad \sigma_{\gamma \gamma}^{H P}=1.87 \times 10^{-6} s^{\varepsilon_{0}} \tag{23}
\end{gather*}
$$

To get $\sigma_{r r}$ we only need is to add the contributions from $f^{\prime}, A$ and the quark box diagram ( ${ }^{\sigma q}$ ). But the coupling constants of $f^{\prime}$ and $A$ are related to that of $f$ by $[16]$

$$
\begin{equation*}
g_{f \gamma r}^{2}: g_{A \gamma \gamma}^{2}: g_{f^{\prime} \gamma \gamma}^{2}=25: 9: 2 \tag{24}
\end{equation*}
$$

These three Reggeons give a cross section of $1.44 \sigma_{r r}^{f}$. Therefore, we finally get:

$$
\begin{equation*}
\sigma_{r \gamma}=\sigma_{q q}+1.44 \sigma_{r \gamma}^{f}+\sigma_{r \gamma}^{P}+\sigma_{r \gamma}^{H P} \tag{25}
\end{equation*}
$$

## Discussion

We notice that our expression in eq. (20) for $\sigma_{p}$ is in good agreement with the data as given in fig. (3). Moreover, the coefficients in eq. (20) are nearly equal to those of Donnachie and Landshoff in eq. (21). Furthermore, the values of $\sigma_{\gamma \gamma}$ in eq. (25) are consistent with our previous results ${ }^{[14]}$. We notice that if we use

$$
\begin{equation*}
\sigma_{\gamma \gamma}=\frac{\sigma_{p p}^{2}}{\sigma_{p p}}=\frac{\left(117.7 \times 10^{-3} s^{-R}\right)^{2}}{46.55 s^{-R}} \tag{26}
\end{equation*}
$$

we get the value of $297.6^{\times 10^{-6}} s^{-R}$ for $\sigma_{\gamma r}$ in the low energy region which is nearly equal to $1.44 \sigma_{r r}^{f}$. However, the value in eq. (25) is consistent with Regge model while that of ${ }^{p p}$ in eq. (26) is the net contribution from $\omega-f$ mesons. Finally we should notice that the coupling constant of the soft pomeron with quarks is higher than that of the hard pomeron. This is evident by calculating the values of $\beta$ and $\beta_{H}$ from $\quad \sigma_{p p}$. These values are $\beta=1.64 m b^{\frac{1}{2}}$ and $\beta_{H}=0.037 m b^{\frac{1}{2}}$

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Figure (1a) schematic diagram of photon -proton total cross section


Figure (1b) the cross section in terms of the imaginary part of the elastic amplitude


Figure (2) Regge representation for photon -proton elastic amplitude. The dashed line represents the exchanged Reggeon or pomeron. The triangle represents their coupling with photons.

# المثطع العرضـي الكلي لنصادم الفوتون مع البروتون مـع عملية الثّفكيّ خالا محمد عبد الحمد <br> $$
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$$ 

تم استخدام نموذج رجي لإيجاد ملائمة لنتائج المقطع العرضي الكلي للفوتون مع البروتون • و كالمعتاد فأن الملائمة تعتمد على حساب ثوابت الازدواج للرجونات و البومرونات المتبادلة في العطلية . يتضمن النموذج ثلاث مكونات حسب نموذج رجي حيث يسيطرالرجون f على منطقة الطاقة الواطئة وفي الطاقات العالية فتسيطر الرجونات الناعمة والصلبة ، وبما ان قطب الرجون يمكن تفكيكة فأن المقاطع العرضية تكون مرتبطة بعملية النفكيك. لقد بينا بأن المقع العرضي الكلي لتصادم الفوتون مع الفوتون يمكن ايجادة أيضا بواسطة عملية التفكيك.


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