On Q-Injective, Duo Submodules of C₁-Module
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ARTICLE INFO
Received: 1/3/2021
Accepted: 15/4/2021
Available online: 1/6/2021
DOI: 10.37652/juaps.2022.172431
Keywords:
Quasi-injective module,
Duo submodule, Stable module,
Pseudo-injective module.

ABSTRACT
This note investigates modules having quasi-injective and duo submodules. We introduce a new generalization of C₁-module. The main method that was adopted in this generalization is how to obtain a submodule N in M having the characteristic Quasi-injective. We investigate the relationship between pseudo-injective module and Quasi-injective property of C₁-module. Finally, we introduce a new relationship between Quasi-injective submodule and anti-hopfian module.

1. INTRODUCTION
All the modules in this paper have a unity. Many searchers studied Quasi-injective and injective modules in details. Here we study Quasi-injective of any submodule N of M. In [1], An R-module P is a projective module if there exists an R-module Q such that P ⊕ Q is a free R-module; also more details about injective and projective module can find it in same reference. In [2], we can find the definition of a Quasi-injective module (briefly Q-injective). Also, in [3], the author said M is pseudo-injective module (p-injective module) if ∀ N ≤ M, each R-isomorphism g: N → M can be extended to an R-endomorphism of M. In [4], A module M is called uniform if N₁ and N₂ are non-zero submodules of M; N₁ ∩ N₂ ≠ 0 the intersection of any two non-zero submodules is nonzero, equivalently, M is uniform if 0 ≠ N ≤ ess M. In [5], N ≤ M is called stable if for each R-homomorphism f: N → M implies f(N) ⊆ N, and an R-module M is called fully stable in case every submodule of M is stable.

In this article, we investigate some facts about any submodule N of C₁-module M like Q-injective and duo properties. Also we use other properties in order to satisfy the same goal such as hopfian, anti-hopfian and self-injective modules.

2. PSEUDO-INJECTIVE and QUASI-INJECTIVE SUBMODULES
In this section, we will study two important properties of submodule N of M namely Quasi-injective and P-injective. Via this submodule, we obtain a new characterization of C₁-module. Moreover, we should provide another property namely fully invariant of this submodule. Note that Q-injective itself injective.

Definition 2.1. [1]. An R-module M is called injective if for every monomorphism h: M₁ → M₂ and homomorphism f: M₁ → M₃ there exists a homomorphism g: M₂ → M₃ such that g ∘ h = f.

Definition 2.2. [2]. Let M be an R-module. Then M is said to be Q-injective if for each submodule N of M and R-homomorphism f: N → M can be extended to an R-endomorphism of M.

Definition 2.3. [3]. An R-module M is called pseudo-injective, if for every submodule N of M, each R-
isomorphism \( g: \mathcal{N} \to \mathcal{M} \) can be extended to an \( R \)-endomorphism of \( \mathcal{M} \).

**Lemma 2.4.** [6]. Let \( \mathcal{M} \) be an \( R \)-module over P.I.D. If \( \mathcal{M} \) is pseudo-injective module, so it is a Q-injective.

Now we need to find a submodule \( \mathcal{N} \) of \( \mathcal{M} \) such that \( \mathcal{N} \) is a Q-injective with invariant property. From [3], any pseudo-injective module over P.I.D is a Q-injective; this means if \( \mathcal{M} \) is a module on P.I.D, so \( \mathcal{N} \leq \mathcal{M} \) on P.I.D, but \( \mathcal{M} \) is pseudo-injective; \( \mathcal{N} \) is a pseudo-injective and hence \( \mathcal{N} \) is a Q-injective.

Note that to understanding lemma (2.4), we can see [7].

The following theorem explain the relationship between pseudo-injective and \( C_1 \)-module over P.I.D.

**Theorem 2.5.** Let a ring \( R \) be a P.I.D If \( \mathcal{M} \) is a pseudo-injective \( C_1 \)-module over \( R \), then any submodule \( \mathcal{N} \leq \mathcal{M} \) is a Q-injective and \( f(\mathcal{N}) \subseteq \mathcal{N} \); so \( \mathcal{M} \) is Q-injective-duo-\( C_1 \)-module.

**Proof:** Suppose that a module \( \mathcal{M} \) is pseudo-injective. Let us take \( \mathcal{N} \leq \mathcal{M} \). We have \( \mathcal{M} \) any module on P.I.D So also \( \mathcal{N} \leq \mathcal{M} \) on P.I.D. But \( \mathcal{M} \) is pseudo-injective, then \( \mathcal{N} \) is pseudo-injective over P.I.D. Hence \( \mathcal{N} \) is Q-injective with \( f(\mathcal{N}) \subseteq \mathcal{N} \) imply \( \mathcal{N} \) is fully invariant (duo submodule of \( \mathcal{M} \). Thus \( \mathcal{M} \) is Q-injective-duo-\( C_1 \)-module.

Now we introduce another way to obtain any submodule \( \mathcal{N} \) of \( C_1 \)-module \( \mathcal{M} \) and be Q-injective. This way depends on new domain namely Dedekind domain. (\( R \) is a Dedekind domain if it is integrally closed, Noetherian and if \( 0 \neq p \) is a maximal; \( p \) is prime ideal). So if \( R \) is a Dedekind domain, then it is a UFD if and only if \( R \) is P.I.D. See the next Lemma:

**Lemma 2.6.** [7]. Let \( \mathcal{M} \) be an \( R \)-module over Dedekind domain. If \( \mathcal{M} \) is pseudo-injective (P-injective), then \( \mathcal{M} \) is a Q-injective and so \( \mathcal{N} \leq \mathcal{M} \) is a Q-injective submodule.

**Theorem 2.7.** Let \( \mathcal{M} \) be a Pseudo-injective- \( C_1 \)-module over Dedekind domain. If \( \mathcal{M} \) is stable, then \( \mathcal{M} \) is Q-injective -duo-\( C_1 \)-module.

**Proof:** Assume that a module \( \mathcal{M} \) is Pseudo-injective and \( R \) is a Dedekind domain. From lemma (2.6), \( \mathcal{M} \) is a Q-injective. So \( \mathcal{N} \leq \mathcal{M} \) is also Q-injective. But \( \mathcal{M} \) is stable, so \( \mathcal{N} \) is a fully invariant. Therefore \( \mathcal{N} \) is a duo submodule of \( \mathcal{M} \).

**Lemma 2.8.** [7]. Let \( \mathcal{M} \) be an \( R \)-module. If the following statements are true:

1. \( \mathcal{N} \) is a Multiplication ring;
2. \( \mathcal{M} \) is P-injective;
3. \( T(\mathcal{M}) = \mathcal{M} \);

then \( \mathcal{M} \) is Q-injective and so \( \mathcal{N} \) is also Q-injective.

**Theorem 2.9.** Let \( \mathcal{M} \) be a module over a ring \( R \). If:

1. \( \mathcal{M} \) is a multiplication ring;
2. \( T(\mathcal{M}) = \mathcal{M} \);
3. \( \mathcal{M} \) is stable;
4. \( \mathcal{M} \) is \( D_1 \)-module and Pseudo-injective;

then \( \mathcal{M} \) is Q-injective-duo-\( C_1 \)-module.

**Proof:** Assume that \( T(\mathcal{M}) = \mathcal{M} \) and \( R \) is a multiplication ring. Then from [8], \( T(\mathcal{N}) = \mathcal{N} \) (any submodule of torsion module is torsion). Since \( \mathcal{M} \) is P-injective, then \( \mathcal{M} \) is a Q-injective and hence \( \mathcal{N} \) is P-injective and \( T(\mathcal{N}) = \mathcal{N} \). Hence \( \mathcal{N} \) is a Q-injective. Since \( \mathcal{M} \) is stable, then \( \mathcal{N} \) is a fully invariant. But from condition (4), \( \mathcal{M} \) is \( C_1 \)-module. Then \( \mathcal{M} \) is a Q-injective-duo-\( C_1 \)-module.

**Corollary 2.10.** If \( \mathcal{M} \) is a \( C_1 \)-pseudo-injective R-module, then \( \mathcal{M} \) is a Q-injective-duo-\( C_1 \)-module, knowing that \( f(\mathcal{N}) \subseteq \mathcal{N} \) and \( T(\mathcal{M}) = \mathcal{M} \).

Recall that any \( R \)-module \( \mathcal{M} \) is called nonsingular if, for all \( m \in \mathcal{M} \) with \( r(m) \leq \text{ess} \) \( R \) implies that \( m = 0 \). Or \( Z(\mathcal{M}) = \{ x \in \mathcal{M}; \exists \text{ a right ideal } I \text{ of } R \text{ such that } I \leq \text{ess} \text{ and } X = 0 \} \) and \( Z(\mathcal{M}) = 0 \) [9].

**Lemma 2.11.** If \( \mathcal{N} \leq \text{ess} \mathcal{M} \) and \( Z(\mathcal{M}) = 0 \) in pseudo-injective module \( \mathcal{M} \), then \( \mathcal{N} \) is Q-injective.

**Proof:** Let \( \mathcal{N} \leq \text{ess} \mathcal{M} \) and \( Z(\mathcal{M}) = 0 \). Let \( g: \mathcal{N} \to \mathcal{M} \) be an \( R \)-homomorphism. So \( \text{Ker}(g) = 0 \) or \( \text{Ker}(g) = \mathcal{N} \). Suppose that \( \text{Ker}(g) = \mathcal{N} \), so \( g \) can be extended to homomorphism \( h: \mathcal{M} \to \mathcal{M} \). Now if \( \text{Ker}(g) = 0 \), so \( g \) is one to one and can be extended to \( R \)-homomorphism from \( \mathcal{M} \to \mathcal{M} \) (\( \mathcal{M} \) is Pseudo-injective). Hence \( \mathcal{N} \) is Q-injective.

**Corollary 2.12.** Let \( \mathcal{M} \) be a \( C_1 \)-pseudo-injective R-module. If \( f(\mathcal{N}) \subseteq \mathcal{N} \), \( \mathcal{N} \leq \text{ess} \mathcal{M} \) and \( Z(\mathcal{M}) = 0 \); then \( \mathcal{M} \) is Q-injective-duo-\( C_1 \)-module.

Now we present another way in order to obtain that any submodule \( \mathcal{N} \leq \mathcal{M} \) is a Q-injective. But before that we need to present some important definitions that are closely related to the mentioned way. Firstly, a concept of Stable-Q-injective was explained in [6]. Let \( \phi: \mathcal{N} \to \mathcal{M} \) and \( \phi(\mathcal{N}) \subseteq \mathcal{N} \). Then \( \mathcal{M} \) is called stable module. So if every \( \mathcal{N} \leq \mathcal{M} \) is stable this means \( \mathcal{M} \) is fully stable module (F-stable).

If \( \mathcal{N} \leq \mathcal{M} \) is stable and can be extended R-homomorphism \( (\mathcal{N} \to \mathcal{M}) \) to an \( R \)-endomorphism \( (\mathcal{M} \to \mathcal{M}) \), then \( \mathcal{M} \) is called stable-Q-injective R-module. Also, If \( R \) is an integral domain and \( \mathcal{M} \) is an R-module, then an element \( x \in \mathcal{M} \) is called torsion element if \( \exists 0 \neq r \in R \exists rx = 0 \). [10]. So we define:

\[
T(\mathcal{M}) = \{ x \in \mathcal{M}; x \text{ is a torsion element} \}.
\]
Note that:

1. If \( T(M) = M \), then a module \( M \) is called torsion-module.
2. If \( T(M) = 0 \), then a module \( M \) is called torsion-free-module.

**Lemma 2.13.** [6]. Let \( M \) be a stable-Q-injective \( R \)-module. If \( M \) is an injective \( R \)-module, then it is \( Q \)-injective.

**Theorem 2.14.** Let \( M \) be a \( C_1 \)-module. If \( M \) is a F-stable and stable-
Q-injective; then \( M \) is \( Q \)-injective-duo-\( C_1 \)-module.

**Proof:** Let \( N \subseteq M \) and let \( \varphi : N \to M \) be an \( R \)-homomorphism of \( M \). So \( N \) is a stable because \( M \) is a \( F \)-stable. But from stable-
Q-injective of \( M \), there is an \( \varphi : M \to M \) \( \varphi \) extends \( \varphi \). Hence \( M \) is a \( Q \)-injective. Thus \( M \) is \( Q \)-
injective-duo-\( C_1 \)-module.

**Corollary 2.15.** Let \( M \) be a \( C_1 \)-module. If \( N \subseteq M \); \( \varphi(N) \subseteq N \) \( \exists \varphi : N \to M \) be a homomorphism and \( M = M_1 \oplus M_2 \) is a stable-
Q-injective, then \( M \) is \( Q \)-injective-duo-\( C_1 \)-module.

**Proof:** By Theorem (2.14).

**Remark 2.16.** From definition of fully invariant submodule and definition of stable, we find the two meanings are same.

Recall that a ring \( R \) is called Quasi-Frobenius (QF-ring) if every projective module is injective; or every injective module
is discrete. From [11], every projective-module is injective and then every injective-module is \( Q \)-injective.

**Corollary 2.17.** Let \( M \) be a \( C_1 \)-module over QF-ring. If \( M \) is a projective module and stable in \( R \), then \( M \) is \( Q \)-injective-
duo-\( C_1 \)-module (\( N \) is \( Q \)-injective submodule).

**Proof:** Let \( R \) be a QF-ring. Since \( M \) is a projective \( R \)-module, then \( M \) is an injective module and hence \( Q \)-injective.
Then every submodule \( N \) of \( M \) is \( Q \)-injective. Note that \( M \) is stable module; so for \( \varphi : N \to M \) be a homom. we get \( \varphi(N) \subseteq N \). Thus \( M \) is \( Q \)-injective-duo-\( C_1 \)-module.

Recall that a module \( M \) is called \( D_1 \)-module if for \( N < M \), \( \exists M = M_1 \oplus M_2 \) is a coessential sub of \( N \); or if \( N, K \leq M \) and \( H \leq N \), then \( M = H \oplus K \) and \( N \cap H \leq M \). So \( D_1 \)-
module is extending.

**Proposition 2.18.** Let \( M \) be an \( R \)-module over QF-ring \( R \). If:

1. \( M \) is \( D_1 \)-module;
2. \( M \) is stable module;
3. \( M \) is a free-module;
then \( M \) is \( Q \)-injective-duo-\( C_1 \)-module.

**Proof:** From condition (1); \( M \) is \( C_1 \)-module. From condition (2); \( \exists \) an \( R \)-homomorphism \( \varphi : N \to M \) \( \exists \varphi(N) \subseteq N \) \( (N \) is fully invariant). So \( N \) is a
duo submodule. Condition (3); gives \( M \) is a free-module. So if we take \( F \) is a free-
R-module

on a set \( S \). Suppose that \( N_1, N_2 \) two modules over the ring \( R \). Let \( \varphi : N_1 \to N_2 \) is a homomorphism.

\[ \forall x \in S; \text{ we choose } a_x \in N_1 \exists j(x) = a_x \ldots \ldots \ (1) \]

Also,

\[ \forall x \in F, g(x) \in N_2 \text{ and } \varphi : N_1 \to N_2 \text{ is onto.} \]

Then \( \exists a_x \in N_1 \exists \varphi(a_x) = g(x) \ldots \ldots \ (2) \)

Since \( F \) is a free-R-module on \( S \), \( \exists \) a unique homomorphism \( h : F \to N_1 \exists h \circ i = j \ldots \ldots \ (3) \)

To prove that \( \varphi \circ h = g \). Let \( x \in F \). So

\[ x = \sum r_k x_k; x_k \in S, r_k \in R; k = 1, 2, \ldots, n \]

(because \( F \) is generated by \( s \in F = \langle s \rangle \)).

Now

\[ (\varphi \circ h)(x) = (\varphi \circ h)(\sum r_k x_k) \]

\[ = \varphi(h(\sum r_k x_k)) \]

\[ = \varphi(\sum r_k h(x_k)), \]

\( h \) is homomorphism.

\[ = \varphi(\sum r_k (h(i(x_k)))). \]

Now

\[ (\varphi \circ h) = \varphi(\sum r_k ((h \circ i)(x_k))). \]

\[ = \varphi(\sum r_k j(x_k)); \ (by \ (3)). \]

\[ = \varphi(\sum r_k a_{x_k}); \ (by \ (1)). \]

\[ = \sum r_k \varphi(a_{x_k}); \varphi \text{ homomorphism.} \]

\[ = \sum r_k g(x_k); \ (by \ (2)). \]

\[ = g(\sum r_k x_k). \]

g homomorphism. So \( \varphi \circ h = g \). Thus \( M \) is a projective and hence \( M \) is injective \( (M \) is a \( Q \)-injective). Then \( N \subseteq M \) is \( Q \)-injective. Thus \( M \) is a \( Q \)-injective-
duo-\( C_1 \)-module.

**Lemma 2.19.** For a ring \( R \), we have \( R_g \) is a semi-simple if and only if \( R \) is a semi-simple and so any module \( M \) over \( R \) is a semisimple module.

**Proof:** We need to prove the following.

1. \( R_g \) is semisimple if and only if \( R \) is semisimple.
2. \( M \) is a semisimple module over \( R \).

From [11], we can get the proof of (1).

Now we need to proof (2):

If \( R_g \) is a semi-simple and if \( M = M_g \exists m \in M \), then \( R \) is a
semi-simple as an epimorphic image of \( R_g \). So

\[ M = \sum mR, m \in M \] as a sum of semi-simple module is again

**Lemma 2.20.** Let a ring \( R \) be a semi-simple, and \( M \) be an \( R \)-
module. Then every submodule \( N \subseteq M \) is \( Q \)-injective.
Proof: Since $R$ is a semisimple ring, then every module $\mathcal{M}$ over $R$ is a semisimple. So $\mathcal{N} \leq \mathcal{M}$ is a direct summand. Hence $\mathcal{M}$ is injective $R$-module. But every injective $R$-module is a $Q$-injective. Thus $\mathcal{N}$ is $Q$-injective.

**Theorem 2.21.** Let $R$ be a semisimple ring and $\mathcal{M}$ is an $R$-module. If $\mathcal{M}$ is $D_1$-module and stable; then it is $Q$-injective-duo-$C_1$-module.

**Proof:** It is clear that from lemma (2.20), $\mathcal{N} \leq \mathcal{M}$ is $Q$-injective. But $\mathcal{M}$ is a stable, then $\exists f: \mathcal{N} \rightarrow \mathcal{M} \ni f(\mathcal{N}) \leq \mathcal{N}$. So $\mathcal{N}$ is a fully invariant and hence $\mathcal{M}$ is a duo ($\mathcal{N}$ is a duo submodule). We have $\mathcal{M}$ is $D_1$-module. So it is $C_1$-module. Thus $\mathcal{N}$ is $Q$-injective of $\mathcal{M}$.

**Corollary 2.22.** Let $\mathcal{M}$ be an $R$-module. If:

1. $\mathcal{M}$ is projective module;
2. $\mathcal{M}$ is a simple module;
3. $\mathcal{M}$ is $Q$-injective;

then $\mathcal{N}$ is $Q$-injective and duo submodule of $C_1$-module.

**Proof:** It is clear that projective module means $C_1$-module. Also, if a module $\mathcal{M}$ is simple, then $\mathcal{M}$ is duo-module. ($\mathcal{N} \leq \mathcal{M} \ni \mathcal{N}$ is fully invariant; $f(\mathcal{N}) \leq \mathcal{N}$ and $f: \mathcal{N} \rightarrow \mathcal{M}$ is an $R$-homomorphism). Now from condition (3), we have $\mathcal{M}$ is Quasi-projective. So $\mathcal{M}$ is a $Q$-injective and hence $\mathcal{N}$ is a $Q$-injective of $C_1$-module.

Recall that any ring $R$ is called $V$-ring if every simple $R$-module is injective [12].

**Corollary 2.23.** Let $\mathcal{M}$ be a $D_1$-$R$-module over $V$-ring. Then $\mathcal{M}$ is $Q$-injective-duo-$C_1$-module.

### 2. HOPFIAN, SELF-INJECTIVE MODULES AND Q-INJECTIVE SUBMODULE

From [13], a module $\mathcal{M}$ is called self-p-injective if $\mathcal{M}$ satisfy the following condition; every homomorphism from a projection invariant submodule of $\mathcal{M}$ to $\mathcal{M}$ can be lifted to $\mathcal{M}$.

(\#) Every self-injective is injective module.

**Definition 3.1.** Any $R$-module $\mathcal{M}$ is called indecomposable if $\mathcal{M}$ has no proper non trivial complement submodule $\mathcal{M}_1$ ($\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2$, so $\mathcal{M}_1 = 0$ or $\mathcal{M}_1 = \mathcal{M}$).

**Example 3.2.** $Z$ is indecomposable $Z$-module, but $Z$ is not simple $Z$-module ($Z$ contains proper submodule $2Z$).

Therefore, every simple module is indecomposable, but the converse is not true.

**Theorem 3.3.** Let $\mathcal{M}$ be an indecomposable self-P-injective $R$-module. Then any $C_1$- module is $Q$-injective-duo-$C_1$-module.

**Proof:** From definition of self-p-injective, there exists $\mathcal{K}$ submodule of $\mathcal{M}$ such that $\mathcal{K}$ is fully invariant. Assume that $\mathcal{M}$ is indecomposable module, so every submodule of $\mathcal{M}$ is projective invariant. Then $\mathcal{M}$ is $Q$-injective. Thus $\mathcal{M}$ is $Q$-injective-duo-$C_1$-module.

Recall that any module $\mathcal{M}$ is called Hopfian if every surjective $f$ in $\text{End}(\mathcal{M})$ is isomorphism and a non simple module is called anti-Hopfian if proper submodule of $\mathcal{M}$ is a non-Hopfian kernel such that a submodule $\mathcal{N}$ of $\mathcal{M}$ is non-Hopfian kernel (for $\mathcal{M}$) if there exists an isomorohism $\mathcal{M}/\mathcal{N}$ to $\mathcal{M}$ [14]. Or an $R$-module $\mathcal{M}$ is anti-Hopfian if $\mathcal{M}$ is non simple and all nonzero factor modules of $\mathcal{M}$ are isomorphic to $\mathcal{M}$; that is for all $\mathcal{N} \leq \mathcal{M}$, $\mathcal{M}/\mathcal{N} \cong \mathcal{M}$ [16].

**Example 3.4.** Any module of semisimple Artinian ring with finite length is Hopfian module.

**Lemma 3.5.** Let $\mathcal{M}$ be an $R$-module. If $\mathcal{M}$ is anti-Hopfian, then every submodule $\mathcal{N}$ of $\mathcal{M}$ is $Q$-injective [15].

**Theorem 3.6.** Let $\mathcal{M}$ be $C_1$-$R$-module. If $\mathcal{M}$ has exactly one non-zero proper submodule and $\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2 \ni \mathcal{M}_1, \mathcal{M}_2$ are simple modules, then $\mathcal{N} \leq \mathcal{M} \ni \mathcal{N}$ is a $Q$-injective of $\mathcal{M}$.

**Proof:** From [14], $\mathcal{M}$ is anti-Hopfian module. Since $\mathcal{M}_1$ and $\mathcal{M}_2$ are simple modules, then $\mathcal{M}$ is a simple module and so it is a duo module ($\mathcal{N}$ is a duo submodule). From lemma (3.5), the proof is completed.

**Corollary 3.7.** Let $R$ be a Dedekind domain, and $\mathcal{M}$ is $C_1$-module with $\text{Rad}(\mathcal{M}) \neq \mathcal{M}$. If $\mathcal{M} \cong R/I^2 \ni 1$ is a non-zero ideal of $R$ and $\mathcal{N}$ is duo submodule of $\mathcal{M}$, then $\mathcal{N} \leq \mathcal{M}$ is $Q$-injective in $C_1$-module.

**Proof:** From [14] and Lemma (3.5).

### 3. CONCLUSIONS

This paper investigated modules having a submodule are duo and Quasi-injective properties. Tow generalization of $C_1$-module have been studied. We proved that any module has pseudo-injective, $\mathcal{N}$ is essential in $\mathcal{M}$ and stable, this mean $\mathcal{M}$ is a Quasi-injective-duo-$C_1$-module where $R$ is a Dedekind domain. Also same goal can obtained it if $\mathcal{M}$ is a projective and stable with $\mathcal{N}$ is an essential in $\mathcal{M}$.

### REFERENCES


C1 حول المقاسات الجزئية شبه الغامرة والثنائية للمقاس من نوع
عبد السلام فائق طالك* و ماجد محمد عبد
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الخلاصة:
في هذا البحث تم تحقيق المقاس الذي يمتلك مقاسات جزئية شبه غامرة وثنائية. قمنا تعميم جديد للمقاس من نوع C1 باستخدام الطريقة الرئيسية التي اعتمدت على كيفية الحصول على مقاس جزئي N في المقاس M، وكان هذا المقاس C1 للمقاسا الأسلي للتقاطع بين المقاسات الجزئية شبه الغامرة والمقاس الغامر الكاذب. 

anti-hopfian