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Developed Protocol for Key Exchange Based on Irreducible Polynomial

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Abstract: The Aim of this paper is to design a protocol for key exchanging to work on the available computers for different data security application. This paper proposed idea to modify the Diffie-Hellman key exchange by using truncated polynomial instead of discrete logarithm problem to overcome the problem of large prime numbers and non-full coverage of the finite set. The proposed method depends on the arithmetic polynomials. The Irreducible truncated polynomial mathematics is highly efficient and compatible with personal computers.

Keywords: key exchange, Irreducible Polynomial, Diffie-Hellman

Introduction

Key generation is the process of generating keys for cryptography. A key is used to encrypt and decrypt whatever data is being encrypted/decrypted. Modern cryptographic systems include symmetric-key algorithms (such as DES and AES) and public-key algorithms (such as RSA)[12]. Symmetric-key algorithms use a single shared key; keeping data secret requires keeping this key secret. Public-key algorithms use a public key and a private key. The public key is made available to anyone (often by means of a digital certificate). A sender will encrypt data with the public key; only the holder of the private key can decrypt this data[11].

The key exchange problem is how to exchange whatever keys or other information are needed so that no one else can obtain a copy. With the advent of public key / private key cipher algorithms, the encrypting key (public key) could be made public, since (at least for high quality algorithms) no one without the decrypting key (the private key) could decrypt the message[1,13].

The key exchange protocols can be categorized in to two categories: unauthenticated and authenticated. In an unauthenticated key exchange, the parties involved in key exchange do not authenticate each other. As a result, the adversary can launch a man-in-middle attack to agree upon a different key with each of the party involved in key exchange. This situation is resolved by using authenticated key exchange. The protocol is authenticated if every party verifies the authenticity of all the other parties involved in the key exchange [6, 11].

Diffie-Hellman Key Exchange

The Diffie-Hellman key agreement was invented in 1976 during collaboration between Whitfield Diffie and Martin Hellman and was the first practical method for establishing a shared secret over an unprotected communications channel [7]. Diffie-Hellman depend on discrete logarithm problem and prime number. The Diffie-Hellman Key Exchange's security depends on the difficulty of solving the discrete log problem, since no fast algorithms have been discovered for these problems; the DH Key Exchange is still in use today and is thought to be one of the most secure methods available [8].

In the following review, different methods are modified of Diffie-Hellman:

Bhattacharya P., Debbabi M. and et al, (2005), proposed two modifications of Diffie-Hellman. The first modification is to change the domain to integer with \( n=2pt \) where \( Z^{*n} \) is still cyclic and the second modification is to change the domain to Gaussian arithmetic \( Z^{*n} \)[i]. After implementing the three algorithms they found that the symmetric key size derived from the two modified algorithms is much greater than the classical one. Moreover, attacking the two modified algorithms using Pohlig-Hellman algorithm, using the same prime value \( p \) and private value \( a \) or \( b \), needs much more time than the classical one [5].

Also Pathak H. K., Manju S. (2010) proposed a new public key cryptosystem and a Key Exchange Protocol based on the generalization of discrete logarithm problem using Non-abelian group of block upper triangular matrices of higher order. The proposed cryptosystem is efficient in producing keys of large sizes without the need of large primes. The
security of both the systems relies on the difficulty of discrete logarithms over finite fields [10].

**Irreducible polynomial**

In mathematics, the adjective irreducible means that an object cannot be expressed as the product of two or more non-trivial factors in a given set [2]. For any field $F$, the ring of polynomials with coefficients in $F$ is denoted by $F[x]$. A polynomial $p(x)$ in $F[x]$ is called irreducible over $F$ if it is non-constant and cannot be represented as the product of two or more non-constant polynomials from $F[x]$. The property of irreducibility depends on the field $F$; a polynomial may be irreducible over some fields but reducible over others [3, 4].

**Proposal method**

This paper produce a modified Diffie-Hellman key exchange by using truncated polynomial instead of discrete logarithm problem and increases the complexity of this method over unsecured channel. There are many conditions before explaining the Proposal Method, these conditions are:

1. Each side (Alice and Bob) has database that contain the value of irreducible polynomial ($g$) and polynomial value ($X$).
2. Both sides must have same database and put their database in a very secret place.
3. The database consist of 360 rows according to the days for year, table (1) shows part from this database.
4. All operations performed to compute the key are applied in a polynomial operations (addition, subtraction, multiplication, and division).

**Key generation**

two sides want to exchange key (Alice and Bob) as following:
1. Alice selects two integer numbers ($A_1, B_1$) and sends the two values to Bob.
2. Bob receive ($A_1,B_1$) and select also two integer ( $A_2, B_2$) and send the two value to Alice.
3. Now each side has ($A_1, B_1, A_2, B_2$) and converts these values to binary then to polynomial.
4. Alice and Bob return values of ($X, g$) from database based on the day and month for communication between them.
5. Each side applying function (1) and (2) to compute ($Y_1, Y_2$):
   
   $Y_1 = (A_1 X B_1) \mod g$. . . . (1)
   $Y_2 = (A_2 X B_2) \mod g$. . . . (2)
6. Now the two sides have two values ($Y_1, Y_2$), each one apply the same function on these values to compute secret key ($Sk$) as:
   
   $Sk = (Y_1 X Y_2) \mod g$. . . . (3)

   Figure (1) that represents the steps of proposed method.

Example 1: Let us take simple example for apply the method when Alice key exchange with Bob in date (1- January):
1. Alice generate two integer values and sends the two values to Bob
   
   $A_1 = 6, B_1 = 4$
2. Bob generate two integer values and sends the two values to Alice
   
   $A_2 = 9, B_2 = 7$
3. Alice and Bob convert each values to binary then polynomials.
   
   $x^2 + x$
   $A_1 = 110 = x^2$
   $B_1 = 100 = x^2 + 1$
   $A_2 = 1001 = x^2 + x + 1$
   $B_2 = 111 = x^2 + x + 1$
4. Both sides return the value ($X$) and ($g$) from database
   
   $X = x^2 + x + 1$
   $g = x^2 + x^3 + x^5 + x + 1$
5. Alice and Bob applied the same functions to find $Y_1, Y_2$

   $Y_1 = (A_1 X B_1) \mod g$
   $Y_1 = (x^2 + x) (x^2 + x + 1) \mod x^2$
   $= [x^2 + x^3 + x + 1] x^2$
   $= [x^2 + x^3 + x + 1] x^2$
   $= [x^2 + x^3 + x + 1] x^2$
   $= [x^2 + x^3 + x + 1] x^2$
   $= [x^2 + x^3 + x + 1] x^2$
   $= [x^2 + x^3 + x + 1] x^2$
   $= [x^2 + x^3 + x + 1] x^2$
   $= [x^2 + x^3 + x + 1] x^2$

   $Y_2 = (A_2 X B_2) \mod g$
   $= (x^8 + 1) x^6 + x + 1 \mod (x^2 + x + 2)$
   $= [x^6 + x^5 + x^4 + x + 1] x^2 + x + 1$
   $= [x^6 + x^5 + x^4 + x + 1] x^2 + x + 1$
   $= [x^6 + x^5 + x^4 + x + 1] x^2 + x + 1$
   $= [x^6 + x^5 + x^4 + x + 1] x^2 + x + 1$

6. Now the sides have two values ($Y_1, Y_2$), each one apply the same function on these values to compute secret key ($Sk$) as:

   $Sk = (Y_1 X Y_2) \mod g$. . . . (3)
Table (1): Simple Part of database.

<table>
<thead>
<tr>
<th>Day-Month</th>
<th>Value ( X )</th>
<th>Irreducible polynomial ( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>$x^5 + x + 1$</td>
<td>$x^9 + x^4 + x^3 + x + 1$</td>
</tr>
<tr>
<td>2-1</td>
<td>$x^5 + x^3 + x^2 + x + 1$</td>
<td>$x^9 + x^4 + x^3 + x + 1$</td>
</tr>
<tr>
<td>3-1</td>
<td>$x^3 + x^5 + x^4 + x^2 + x + 1$</td>
<td>$x^9 + x^4 + x^3 + x + 1$</td>
</tr>
<tr>
<td>4-1</td>
<td>$x^6 + x^5 + x^2 + x + 1$</td>
<td>$x^9 + x^5 + x^3 + x^2 + 1$</td>
</tr>
<tr>
<td>5-1</td>
<td>$x^7 + x^5 + x^3 + x + 1$</td>
<td>$x^9 + x^5 + x^3 + x^2 + 1$</td>
</tr>
<tr>
<td>6-1</td>
<td>$x^8 + x^3 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>7-1</td>
<td>$x^8 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>8-1</td>
<td>$x^8 + x^5 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>9-1</td>
<td>$x^9 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>10-1</td>
<td>$x^9 + x^5 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>11-1</td>
<td>$x^9 + x^5 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>12-1</td>
<td>$x^9 + x^5 + x^2 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>13-1</td>
<td>$x^9 + x^5 + x^2 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>14-1</td>
<td>$x^9 + x^3 + x^3 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>15-1</td>
<td>$x^9 + x^3 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>16-1</td>
<td>$x^9 + x^5 + x$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>17-1</td>
<td>$x^9 + x^5 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>18-1</td>
<td>$x^9 + x^5 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>19-1</td>
<td>$x^9 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>20-1</td>
<td>$x^9 + x^5 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>21-1</td>
<td>$x^9 + x^7$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>22-1</td>
<td>$x^9 + x^8 + x^2 + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>23-1</td>
<td>$x^9 + x + 1$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
<tr>
<td>24-1</td>
<td>$x^9 + x^8 + x^2 + x + x$</td>
<td>$x^9 + x^6 + x^3 + x^2 + x + 1$</td>
</tr>
</tbody>
</table>
Results

The proposed method has been executed on PC computer with “Intel Pentium 4 2.2 MHz Dual Core 2” CPU. The programs implemented using Microsoft Visual Studio 2008 (C#). The result had been compared with the traditional Diffie-Hellman key exchange method and proposed method, Table (2) shows the following results:

<table>
<thead>
<tr>
<th>Systems</th>
<th>Run Time (s:ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffie-Hellman</td>
<td>1:13</td>
</tr>
<tr>
<td>Proposal Method</td>
<td>0:900</td>
</tr>
</tbody>
</table>

Figure (1): proposal method

\[ Y_2 = \frac{x^5 + x^2 + x + 1}{x^7 + x^6 + x^5 + x^2 + x} \]

To find Secret Key (Sk):

\[ Sk = (Y_1 \odot X \oplus Y_2) \mod g \]

\[ x^7 + x^6 + x^5 + x^2 + x \]

\[ (x^5 + x + 1) \] \mod \[ (x^9 + x^4 + x^5 + x + 1) \] \mod

\[ (x^{12} + x^{13} + x^7 + x^4 + x) \]

\[ x^2 + x + 1 \]

\[ (x^9 + x^4 + x^5 + x + 1) \]

\[ x^7 + x^6 + x^5 + x^2 + x \]

(Secret Key)
Analysis

Key randomness for the traditional method is very small when choosing a small prime number; on the other hand the randomness of the new method is always very high whatever the irreducible polynomial because the result is always unexpected. The complexity of the traditional method depends on computing the discrete logarithm which take long time specially when choosing very high prime number, while this method is always complex because it depends on irreducible truncated polynomial. when anyone want to know the key he will try all irreducible polynomial \(( g )\) and Truncated polynomial \(( X )\) on novel function ,so that it became very difficult to conform \(( g )\) with \(( X )\).

Conclusion

this paper present an idea for key exchange protocol by modifying the idea of Diffie-Hellman key exchange and completely remove the discrete logarithm and replace it by irreducible truncated polynomial to overcome the problem of large prime numbers and non-full coverage of the finite set, run time of proposal method less than run time of traditional Diffie-Hellman.

Reference