Approximate Solution of Emden-Fowler Equation Using the Galerkin Method

Asmaa A. Aswhad

Department of Mathematics, College of Education for pure Science Ibn-Al-Haitham, University of Baghdad, Baghdad, Iraq;

A R T I C L E I N F O

Received: 24 / 06 /2023
Accepted: 01 / 06/ 2023
Available online: 08 / 06 / 2023

DOI: 10.37652/juaps.2023.178798

Keywords:
Emden-Fowler Equation,
weighted residual method,
Galerkin method,

A B S T R A C T

The Emden-Fowler equation (E-F.Eq.) used in mathematical with other science like physics, chemical physics and astrophysics, also this equation can be reduces to the Lane–Emden equation with specified function and used it in different sciences with mathematics. Many Authors study analytic and numerical methods to find the solution for this kind of the equations in the case linear or nonlinear one of these methods the homotopy-perturbation method.

In this work the approximate solution for generalized (E-F.Eq.) in the second order ordinary differential equations was found by Galerkin method which is one of the weighted residual methods and do not need long time also use operator (linear or nonlinear) or differential operator in the any kind of the intervals and compared this solution with the exact solution by discuss the results from applying this method for homogeneous and nonhomogeneous equations and drown the solutions in the same figure to illustrate the results.

1. Introduction:

In mathematics, in the field of applied mathematics, a great interest in numerical solutions emerged in the last century. Many methods appeared to solve initial and boundary value problems that are difficult to find an analytical solution . Some of these methods have gained wide fame either ease of use or for their efficiency compared to similar methods such as iterative methods like in [1], [2], Homotopy-perturbation method [3], Runge- Kutta method [4],Galerkin method[5],[6] which is the subject of study here. We used it to solve the Emden-Fowler equation (E-F.Eq.).

In [7] and [8] study the initial value problems of (E-F.Eq.),

$$y'' + \frac{2}{x}y' + af(x)g(y) = 0,\ x > 0, \text{with} \quad y(0) = \alpha \text{ and } y'(0) = 0, \qquad \ldots(*)$$

where α is a fixed, $f(x)$ and $g(y)$ are given functions.

The problem of find the approximate solution for the equation (∗) and compared that solution with the approximate solution by homotopy-perturbation method studied by [3].

2- Main Notions

u(x) is approximated solution in the weighted-residual method by

$$\tilde{u}(x) \approx U_N(x) = \sum_{j=1}^{N} c_j\phi_j(x) + \phi_0(x)$$

The residual of the approximation is

$$R = S(\sum_{j=1}^{N} c_j\phi_j(x) + \phi_0(x)) - h \neq 0$$

where S is an operator linear or nonlinear.

*Corresponding author at: Department of Mathematics, College of Education for pure Science Ibn-Al-Haitham, University of Baghdad, Baghdad, Iraq;
ORCID:https://orcid.org/0000-0002-1996-9044
;Tel:+964000000000000000
E-mail address: asmaa.a.a@ihcoedu.uobaghdad.edu.iq

The notion in the methods of weighted residuals as follows

$$\int_{\Omega} R(x)W_j dx = 0 \quad (j = 1, 2, ..., n)$$

Anywhere numeral weighted functions W_i = numeral of the constants c_j, [9], [10].

In [6] Abd Almajeed surveys the steps of this method to find the approximate solution of the delay differential equations that in [11].

3- Application Examples

Example (1):

In [12] Ramos gave the exact solution $n = m^4 - m^3$ for the linear, nonhomogeneous (E-F.Eq.)

$$n'' + \frac{8}{w}n' + mn = m^5 - m^4 + 44m^2 - 30m \ , \quad n(0) = 0 \text{ and } n'(0) = 0$$

To find the approximate solution we apply the same steps in [6]

$$R(m) = n'' + \frac{8}{m}n' + mn - m^5 + m^4 - 44m^2 + 30m$$

Suppose $\tilde{n}(m) = a_0 + a_1 m + a_2 m^2$. So from the conditions we get $a_0 = 0$

$\tilde{n}'(m) = a_1 + 2a_2 m$. Also from the conditions we get

$\tilde{n}''(m) = 2a_2$ $\quad a_1 = 0$

$$R(m) = 18a_2 + a_2 m^3 - m^5 + m^4 - 44m^2 + 30m \quad \Rightarrow \quad W = \frac{\partial R}{\partial a_2} = 18 + m^3$$

$$\int_1^{10} WR(m)dm = \int_1^{10} (18 + m^3)\left[18a_2 + a_2 m^3 - m^5 + m^4 - 44m^2 + 30m\right]dm$$

Implies after reduce above integral, $a_2 = \frac{3366889547}{47334880}$

So the approximate solution is

$\tilde{n}(m) = \frac{3366889547}{47334880} m^2$.

Similarly we can apply those steps if we take other intervals.

Example (2):

Apply the steps in [6] to find the approximate solution for the nonlinear, homogeneous (E-F.Eq.)

$$k'' + \frac{8}{m}k' + 18ak + 4ak\ln k = 0 \ , \quad k(0) = 1 \text{ and } k'(0) = 0$$

The exact solution is $k(m) = e^{-am^2}$, [13].

As in [6], the steps for find the approximate solution are

$$R(m) = k'' + \frac{8}{m}k' + 18ak + 4ak\ln k$$

Suppose $\tilde{k}(m) = s_0 + s_1 m + s_2 m^2$. So from the conditions we get $s_0 = 1$

.

Also from the conditions we get $s_1 = 0$

$\tilde{k}'(m) = s_1 + 2s_2 m$ $\quad \tilde{k}''(m) = 2s_2$

$$R(m) = 18s_2 + 18a + 18as_2m^2 + 4a\ln(1+s_2m^2) + 4as_2m^2\ln(1+s_2m^2)$$

$$W = \frac{\partial R}{\partial s_2} = 18 + 36am + \frac{8am}{(1+s_2m^2)} + 4as_2m^2\frac{2m}{(1+s_2m^2)} + 8am\ln(1+s_2m^2)$$

$$\int_0^1 WR(m)dm = \int_0^1 (18 + 36am + \frac{8am}{(1+s_2m^2)} + 4as_2m^2\frac{2m}{(1+s_2m^2)} + 8am\ln(1+s_2m^2)) \left[18s_2 + 18a + 18as_2m^2 + 4a\ln(1+s_2m^2) + 4as_2m^2\ln(1+s_2m^2)\right]dm$$

Implies after reduce above integral with equal zero right side and assume a=1 we get:

$s_2 = -\frac{145625}{246078}$.

So the approximate solution is $\tilde{k}(m) = 1 + -\frac{145625}{246078} m^2$

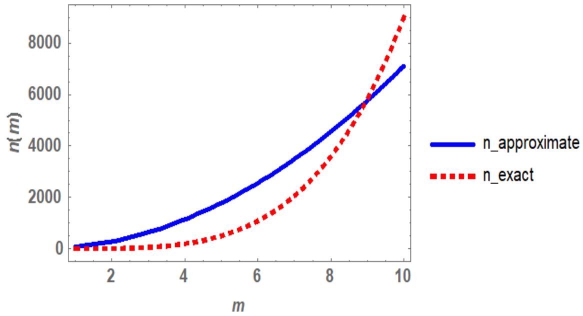
Similarly we can apply that steps if we take other intervals.

4- Discuss the Results and Conclusion

In [3] Chowdhury and Hashim find the approximate solution for the above examples but not drawn these solutions. In this section the compared between these approximate solutions and exact solution drawn in the same figure.

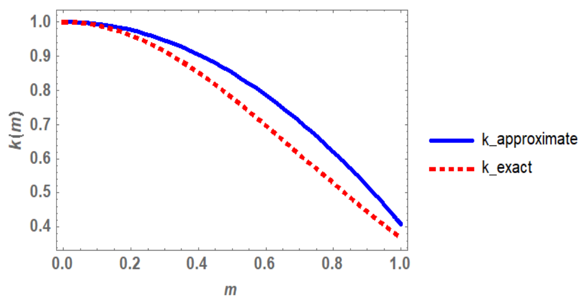
For example (1), the exact solution is $u_0(x) = x^4 - x^3$and the approximate solution is $\tilde{n}(m) = \frac{3366889547}{47334880} m^2$.

So the following figure shown these solutions



For example (2), the exact solution is $k(m) = e^{-am^2}$ and the approximate solution is $\tilde{k}(m) = 1 + -\frac{145625}{246078} m^2$

So the following figure shown these solutions



References:

1. A. A. Aswhad and S. M. Yassein, Sumudu Iterative Method for Solving Nonlinear Partial Differential Equations, Ibn Al-Haitham Jour. for Pure & Appl. Sci.,Vol. 34(2), pp. 23-32, 2021.
2. A. S. Mohammed, Iterative Method for Solving a Nonlinear Fourth Order Integro-Differential Equation, Ibn Al-Haitham Jour. for Pure & Appl. Sci.,Vol. 35(3), pp. 206–212, 2022.
3. M.S.H. Chowdhury and I. Hashim, Solutions of Emden–Fowler equations by homotopy-perturbation method, Nonlinear Analysis: Real World Applications, Vol.10, pp.104–115, 2009.
4. Sh. J. Mohammed and M. A. Mohammed, Runge-Kutta Numerical Method for Solving Nonlinear Influenza Model, Ibn Al-Haitham Int. Con. for Pure and App.Sciences (IHICPS), Jou. of Phy. Con. Series 1879(2021)032040,doi:10.1088/1742-6596/1879/3/032040.
5. J. A. Al-Hawasy, The Approximation Solution of a Nonlinear Boundary Value Problem Via Galerkin Finite Elements Method with Crank-Nicolson, Ibn Al-Haitham Jour. for Pure & Appl. Sci., Vol. 31(3), pp. 126-134, 2018.
6. S. H. Abd Almjeed, Compare Between Least Square Method and Petrov-Galerkin Method for Solving the First Order Linear Ordinary Delay Differential Equations, Journal of College of Education for pure sciences(JCEPS), Vol. 7, Number 4, pp. 16-27, December 2017.
7. O.U. Richardson, The Emission of Electricity from Hot Bodies, London, 1921.
8. S. Chandrasekhar, Introduction to the Study of Stellar Structure, Dover, New York, 1967.
9. J.N. Reddy, An introduction to the finite element Method, Third Edition, McGraw-Hill Series in Mechanical Engineering, Copyright © 2006.
10. P.B Bochev,. and M.D. Gunzburger, Finite Element Methods of Least-Square Type, SIAM Review, Vol.40, pp.789-837, 1998.
11. A. A. Aswhad, Approximate Solution of Delay Differential Equations Using the Collocation Method Based on Bernstien Polynomials, Baghdad Science Journal, Vol.8(3), pp.820-825, 2011.
12. J.I. Ramos, Linearization techniques for singular initial-value problems of ordinary differential equations, Appl. Math. Com., Vol.161, pp.525–542, 2005.
13. A.M. Wazwaz, Adomian decomposition method for a reliable treatment of the Emden–Fowler equation, Appl. Math. Comput. Vol. 161, pp. 543–560, 2005.

P- ISSN 1991-8941 E-ISSN 2706-6703 2023,(17), (1):200– 203 *Journal of University of Anbar for Pure Science (JUAPS)* Open Access

حل تقريبي لمعادلة إمدن فاولر باستخدام طريقة جاليركين

أسماء عبد عصواد

قسم الرياضيات، كلية التربية للعلوم الصرفة /ابن الهيثم، جامعة بغداد، بغداد العراق

asmaa.a.a@ihcoedu.uobaghdad.edu.iq

الخلاصة:

معادلة إمدن–فاولر(E–F.Eq) تستخدم في الرياضيات مع العلوم الأخرى مثل الفيزياء والفيزياء الكيميائية والفيزياء الفلكية، كما يمكن اختزال هذه المعادلة إلى معادلة لين–إمدن بدالة محددة واستخدامها في العلوم المختلفة مع الرياضيات. درس العديد من المؤلفين الطرق التحليلية والعددية لإيجاد حل لهذا النوع من المعادلات في الحالة الخطية أو غير الخطية إحدى هذه الطرق وهي طريقة اضطراب التماثل. في هذا العمل تم إيجاد الحل التقريبي لمعادلات نوع إمدن–فاولر المعممة للمعادلات التفاضلية العادية من الدرجة الثانية بواسطة طريقة جاليركين وهي إحدى الطرق المتبقية الموزونة ولا تحتاج إلى وقت طويل وتستخدم العامل (الخطي أو غير الخطي) او العامل التفاضلي ومقارنة هذا الحل مع الحل التام من خلال مناقشة نتائج من خلال تطبيق هذه الطريقة على المعادلات المتجانسة وغير المتجانسة وإدراج الحلول في نفس الشكل لتوضيح النتائج.